SULTAN QABOOS UNIVERSITY
DEPARTMENT OF MATHEMATICS AND STATISTICS
9 March 2008

MATH 2107 CALCULUS I
TEST I VERSION I
(Time allowed: 60 minutes)

NAME:__________________________________**ID#:**_________ **Section:**________

Instructions:

- This test contains 6 pages (3 sheets back to back) and 12 questions. **The empty extra sheet is for rough work and will not be marked**.
- Write your name, ID number and Section number in the first page and ID number at the top of each sheet.
- Attempt all questions, writing your answer in the space below the statement of the question. For questions 1-6 show all your work.
- For **Multiple Choice** questions, **CIRCLE the correct answer**.
- Please do NOT SEPARATE the pages of this booklet.

DO NOT WRITE ON THIS BOX!

Problem	points	score
1	5 pts	
2	4 pts	
3	5 pts	
4	7 pts	
5	4 pts	
6	3 pts	
7-12	12 pts	
TOTAL	40 pts	

1. (a) (2 points) Find $\lim_{x\to 0^+} \frac{x^2-3x+2}{x}$.

$$\lim_{x\to 0^+} \frac{x^2-3x+2}{x} = +\infty.$$

(b) (3 points) Let $f(x) = \begin{cases} \frac{x^2+5x-14}{x^2-4}, & x \neq 2 \\ 3, & x = 2. \end{cases}$

Determine if f is continuous at $x = 2$.

$$\lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{(x+7)(x-2)}{(x-2)(x+2)} = \lim_{x\to 2} \frac{x+7}{x+2} = \frac{9}{4}.$$

$f(2) = 3$. Therefore, f is not continuous at $x = 2$.

2. (4 points) Find $\lim_{x\to 0} \frac{1-\cos 3x}{x^2}$.

$$\lim_{x\to 0} \frac{1-\cos 3x}{x^2} = \lim_{x\to 0} \frac{1-\cos 3x}{x^2} \cdot \frac{1+\cos 3x}{1+\cos 3x} = \lim_{x\to 0} \frac{\sin^2 3x}{x^2(1+\cos 3x)}$$

$$= \lim_{x\to 0} \left(\frac{\sin 3x}{3x}\right)^2 \cdot \frac{9}{1+\cos 3x} = 1 \cdot \frac{9}{2}.$$

3. A car is travelling on a straight road so that in t mins it reaches a distance of $s = 7t^3$ mtrs.

(a) (2 points) What is the average velocity of the car over the first 10 mins of its travel ?

Average velocity of the car over the first 10 minutes is

$$= \frac{s(10) - s(0)}{10 - 0} = \frac{7(10^3) - 0}{10 - 0} = 700 \text{ m/min}$$

(b) (3 points) What is the instantaneous velocity at the end of 5 mins ?

Instantaneous velocity at the end of 5 mins is

$$\lim_{t \to 5} \frac{s(t) - s(5)}{t - 5} = \lim_{t \to 5} \frac{7t^3 - 7(5^3)}{t - 5} = \lim_{t \to 5} \frac{7(t-5)(t^2 + 5t + 25)}{t - 5} = 525 \text{ m/min}$$

4. (a) (3 points) Find $\frac{dy}{dx}$ if $y = x^3 \tan x - \sec 5x + 2$.

$$\frac{dy}{dx} = x^3 \sec^2 x + 3x^2 \tan x - 5 \sec 5x \tan 5x.$$

(b) (4 points) Find an equation of the tangent line to the graph of $y = \frac{x^3 + 7}{x}$ at $x = 1$.

$$\frac{dy}{dx} = \frac{x.3x^2 - (x^3 + 7).1}{x^2} = \frac{2x^3 - 7}{x^2}.$$

Slope at $x = 1$ is -5.

When $x = 1$, $y = 8$.

Therefore, equation of the tangent line to the graph of the function at the point $(1, 8)$ is

$$y - 8 = -5(x - 1)$$

$$\Rightarrow y = -5x + 13.$$

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5. (4 points) Use an appropriate local linear approximation to estimate the value of $(16.1)^{1/4}$.

$f(x) = x^{1/4}, \quad f'(x) = 1/4\, x^{-3/4}, \quad x = 16.1, \quad x_0 = 16.$

Thus the local linear approximation at $x_0 = 16$ is,

$(16.1)^{1/4} \approx 16^{1/4} + \dfrac{0.1}{4}\, 16^{-3/4}$

$\approx 2.0031.$

6. (3 points) Suppose that f is a differentiable function with the property that $f(x+h) = f(x) + f(h) - 7xh$ and $\lim_{h\to 0} \dfrac{f(h)}{h} = 4$. Find $f'(x)$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{f(h) - 7xh}{h} = \lim_{h\to 0} \frac{f(h)}{h} - 7x \lim_{h\to 0} \frac{h}{h} = 4 - 7x.$$

The remainder of this exam consists of **Multiple Choice** questions. **CIRCLE** the correct answer for each question. **No partial credit will be given.** (2 points each)

7. $\lim\limits_{x \to +\infty} \dfrac{3x^5 - 2x^2 + 4}{7x^3 - 2x^5 - 9}$

(A) 0 **(B)** $+\infty$ **(C)** $-\dfrac{3}{2}$ **(D)** $\frac{3}{7}$ **(E)** None of the above

Answer: C

8. If $f(x) = \pi^2 - \pi^3 + \cos\pi - 1$, then $f'(x)$ is equal to

(A) $2\pi - 3\pi^2 - \sin\pi$ **(B)** 0 **(C)** $2\pi - 3\pi^2 + \sin\pi$ **(D)** $2\pi + 3\pi^2 + \sin\pi$
(E) None of the above

Answer: B

9. The graph of the function $f(x) = \sin x$, $-\pi \le x \le \pi$, has horizontal tangent lines at

(A) $x = \pm\dfrac{\pi}{4}$ **(B)** $x = \pm\pi$ **(C)** 0 **(D)** $x = \pm\dfrac{\pi}{2}$ **(E)** None of the above

Answer: D

10. To prove that $\lim_{x\to -1}(9x+1) = -8$, we must show that given any positive number ϵ, we can find a positive number δ such that

$$|x+1| < \delta \qquad \text{implies} \qquad |(9x+1)+8| < \epsilon.$$

To achieve this task, if $\epsilon = 0.05$ then δ can be taken as

(A) $\frac{1}{180}$ **(B)** $\frac{1}{45}$ **(C)** $\frac{2}{45}$ **(D)** $\frac{2}{9}$ **(E)** None of the above

Answer: A

11. Let $y = \cos x$. Then $\frac{d^{50}y}{dx^{50}}$ is equal to

(A) $-50\sin x$ **(B)** $-50\cos x$ **(C)** $\sin^{50} x$ **(D)** $\cos^{50} x$ **(E)** None of the above

Answer: E

12. Let $f(x) = \begin{cases} x, & x \geq 0 \\ x+1, & x < 0. \end{cases}$

Then

(A) f is differentiable everywhere
(B) f is differentiable everywhere except at $x = 0$
(C) f is differentiable everywhere except at $x = 1$
(D) f is differentiable everywhere except at $x = -1$
(E) None of the above

Answer: B

THE END OF THE TEST