Sultan Qaboos University — College of Science
Department of Mathematics and Statistics
MATH2107 — Calculus I
Fall Semester 2006, Interm Test 1
Date: 7 October 2006   Time allowed: 60 minutes

1. [9 marks] Find the following:

   a) $\displaystyle\lim_{x\to+\infty}(\sqrt{x+3}-\sqrt{x})$;   b) $\displaystyle\lim_{x\to1}\frac{x^{3/2}-1}{x-1}$; $\displaystyle\lim_{x\to0}\frac{1-\cos x}{x\sin x}$.

2. [4 marks] Let

$$f(x)=\begin{cases}\sqrt{2x} & \text{for}\quad x>0,\\ \sqrt{-x} & \text{for}\quad x<0.\end{cases}$$

   Use the $\delta-\varepsilon$ definition to prove that $\displaystyle\lim_{x\to0}f(x)=0$.

3. [4 marks] Show that the equation

$$\left\{\sin(\pi x)+(2x+1)\sqrt{x^2+\sin x+1}\right\}^3=0$$

   has at least one solution in the interval $(-1,1)$.

4. [6 marks] Find the values of the constants a and b that make the function

$$f(x)=\begin{cases}4x^2+1, & x\le2,\\ ax+b, & x>2,\end{cases}$$

   differentiable at $x=2$.

5. [10 marks] Let $y=\sqrt{x+4}$.

   a) Find the average rate of change of y with respect to x over the interval $[0,5]$;

   b) Find the instantaneous rate of change of y with respect to x at $x=5$;

   c) Find the equation of the tangent line to the curve at $x=5$;

   d) Using the result of c) as a local linear approximation, find $\sqrt{9.02}$ approximately.

6. [5 marks] Given that $\frac{d}{dx}[f(\tan x)]=x^2\sec^2 x$, find $f'(\tan x)$.

7. [2 marks] Let $f(x)=\frac{x-3}{x-1}$. Show that f is its own inverse.