SQU Department of Mathematics & Statistics
MATH 2108: Calculus II Spring 2010
Homework Assignment

To submit no later than Saturday 8 May, 2010

- This assignment carries a 5% weight of the total course weight
- Present a clear, detailed and thought out work
- Your work must be independently executed
- All cases of plagiarism, if detected, will be dealt with as per university exam regulation
- An entire question or a part of it may be assessed by a quiz during the class

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1. The base of a solid V is the region bounded by $y = \ln x$, $x = 2$ and $y = 0$. Find the volume of this solid if V has the following cross sections perpendicular to the x-axis:
   (a) square cross sections
   (b) semicircular cross sections
   (c) equilateral triangle cross sections.

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2. A swimming pool viewed from above has an outline given by $y = \pm(5 + x)$ for $0 \le x \le 2$. The depth is given by $4 + x$. Compute the volume of the pool.

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3. The shape generated when a circle is rotated around a line is called a *torus*. Use cylindrical shells to compute the volume of the *torus* obtained by revolving the circle $x^2 + y^2 = 4$ about the line $x = 3$.

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4. Evaluate the following:

   (a) $\int \cos(\ln x)\, dx$ (b) $\int \frac{4x+4}{x^4 + x^3 + 2x^2}\, dx$ (c) $\int_{-2}^{0} \frac{3}{\sqrt{-2x - x^2}}\, dx$

   (d) $\int \frac{\sqrt{6x - x^2}}{(x-3)^2}\, dx$ (f) $\int \frac{x^2}{(x^6 - 4)^{3/2}}\, dx$

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5. Use a comparison to determine whether the integral converges or diverges:

   (a) $\int_0^{+\infty} \frac{\sin^2 x}{1+e^x}\, dx$ (b) $\int_2^{+\infty} \frac{x}{x^{3/2} - 1}\, dx$ (c) $\int_1^{+\infty} \frac{x^2 - 2}{x^4 + 3}\, dx$ (d) $\int_0^{\infty} \frac{dx}{\sqrt{x^3 + x}}$

MORE QUESTIONS ON THE NEXT PAGE

6. Theorem 1.4 on page 325 says “Every bounded, monotonic sequence converges.” Now answer each of the following with proper justifications:
   i. Give an example of a bounded, monotonic sequence that converges.
   ii. Give an example of a monotonic sequence that is bounded from above but diverges.
   iii. Give an example of a monotonic sequence that is bounded from below but diverges.
   iv. Give an example of a convergent sequence that is bounded but not monotonic.

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7. Consider the following sequence $\{a_n\}$ for positive integers *n*:

$$a_n = \begin{cases} \frac{1}{2n^2}\cos(\ln n), & \text{if } n \text{ is odd} \\ n^3 \sin^3\left(\frac{1}{\pi n}\right), & \text{if } n \text{ is even} \end{cases}$$

Determine, with justifications, whether or not the sequence converges.

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8. Determine whether the series converges or diverges:

(a) $\sum_{k=1}^{\infty} \sin\left(\frac{1}{k}\right)$

(b) $\sum_{k=1}^{\infty} \left(\sqrt{k^3+5} - k^{3/2}\right)^k$

(c) $\sum_{k=1}^{\infty} \frac{\cos 2k\pi + \sin k\pi}{\sqrt{k} + (5k+3)^2}$

(d) $\sum_{k=1}^{\infty} (-1)^k \left(\ln(k+1) - k^2\right)$

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9. Determine the values of *p* (a real number) for which the series converges:

(a) $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^{1/p}}$ $(p \neq 0)$

(b) $\sum_{k=2}^{\infty} \frac{e^{2pk}}{(2+e^p)^k}$

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