Sultan Qaboos University *Department of Mathematics & Statistics*

Calculus II: MATH 2108 Fall 2009 FINAL EXAM

2 Hrs 30 mins 9 January 2010

- *Write your name, ID and other details on front cover of your answer booklet*
- *Read and focus on the questions very carefully*
- *Answer all 7 (seven) questions*
- *A solution will be deemed complete only if it includes all the necessary steps and the required symbolisms*
- *All questions are self explanatory, so avoid seeking any further clarifications*

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1 [*7 + 7 = 14 Marks*] Evaluate the following integrals:

(a) $\displaystyle\int_0^1 \frac{e^{-x}}{\sqrt{1-e^{-x}}}\,dx$ (b) $\displaystyle\int_0^{\pi/2} (\sin^5\theta)(\sqrt[3]{\cos\theta}\,)\,d\theta$.

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2 (a) [*8 Marks*] Prove that for any positive integer n,

$$\int \cos^n x\,dx = \tfrac{1}{n}\cos^{n-1}x\sin x + \tfrac{n-1}{n}\int \cos^{n-2}x\,dx \quad .$$

(b) [*4 Marks*] Using part (a) or otherwise, evaluate $\int \cos^4 x\,dx$.

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3 (a) [*6 Marks*] The integral $\int_0^1 5\pi\,x(2-x)\,dx$ represents the volume of a solid obtained by revolving a certain region about an axis. Sketch the region and the axis of revolution that produce the solid.

(b) [*6 Marks*] Find the area of the surface obtained by revolving about x – axis the curve $y=\sqrt{1+5x}$ on [0, 2].

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4 (a) [*2 Marks*] State the theorem on *Limit Comparison Test* for series.

(b) [*6 Marks*] Determine, with proper justifications, whether $\displaystyle\sum_{k=1}^{\infty}\frac{2010+5^{-k^3}}{k^2(1+e^{-k^2})}$ is convergent or divergent.

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5 (a) [*3 Marks*] Give an example of a convergent geometric series, and explain why it is convergent.

(b) [*3 Marks*] Does the sequence $a_n=\cos(n\pi)$ converge or diverge? Justify your answer.

(c) [*3 Marks*] Does the series $\sum_{k=0}^{\infty}(-1)^{k+1}\cos(2k\pi)$ converge or diverge? Justify your answer.

(d) [*3 Marks*] Determine whether the series $-1+\frac{1}{3}-\frac{1}{5}+\cdots$ is absolutely convergent or conditionally convergent or divergent. Justify your answer.

TURN THE PAGE OVER FOR MORE QUESTIONS

6 (a) [*8 Marks*] Sketch the polar curves $r = \sin\theta + \cos\theta$ and $r = 2\cos\theta$, indicating the intercepts.

(b) [*2 Marks*] Write down the integral(s) representing the area of the region inside both the curves in part (a). **(DO NOT COMPUTE THE INTEGRAL)**

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7 (a) [*5 Marks*] Show that the power series representation of $f(x) = \tan^{-1}(x^2)$ is given by:

$$\sum_{k=0}^{\infty} (-1)^k \frac{x^{4k+2}}{2k+1}$$

(b) [*5 Marks*] Find the radius of convergence and the interval of convergence of the power series in part (a).

(c) [*2 Marks*] Deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$.

END OF EXAM *TOTAL MARKS = 80*