- *Write your details and <u>section number</u> on the front cover of your answer booklet*
- *Read and focus on the questions very carefully*
- *Answer all 9 (NINE) questions – there are questions on the other side of this page*
- *For full credit all necessary steps must be shown, must be correct and must be written neatly*

1. [*6 Marks*] Compute the volume of the solid generated by rotating about the y – axis the region enclosed by $y = \dfrac{1}{x^2}$, $x = 2$ and $y = 1$.

2. [*6 Marks*] Find the length of the curve $f(x) = \ln(\sec x)$ from $x = 0$ to $x = \frac{\pi}{4}$.

3. [*6 + 8 = 14 Marks*] Evaluate the integrals: (a) $\displaystyle\int \ln(x+1)\,dx$,   ($b$) $\displaystyle\int \frac{\sec^4 \theta}{1 + \tan\theta}\,d\theta$.

4. [*6 Marks*] Use a comparison to determine whether $\displaystyle\int_{0}^{\infty} \frac{1}{x + e^x}\,dx$ converges or diverges.

5. [*2×7 = 14 Marks*] Test the convergence of the following series: (**give the name of the test you use**):

(a) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{\pi k + 2}{7k + 1} \right)^k$ (b) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{2^k k^2}{k!} \right)$.

6. [*7 Marks*] Determine whether the series $\displaystyle\sum_{k=2}^{\infty} (-1)^k \frac{1}{k \ln k}$ is absolutely convergent, conditionally convergent or divergent.

7. Given that $\displaystyle\sum_{k=1}^{\infty} (-1)^{k+1}\frac{(x-1)^k}{k} = \ln(x)$, with interval of convergence $(0, 2]$.

(a) [*2 Marks*] Find the sum of the series $\displaystyle\sum_{k=1}^{\infty} (-1)^{k+1}\left(\frac{1}{k}\right)\left(\frac{1}{2}\right)^k$.

(b) [*2 Marks*] Is it true that $\displaystyle\sum_{k=1}^{\infty} (-1)^{k+1}\frac{2^k}{k} = \ln(3)$? Justify your answer.

(c) [*6 Marks*] Find the power series representation of $f(x) = \dfrac{1}{x}$ about $x = 1$.

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8. Given the Taylor series of $\displaystyle\tan^{-1}x = \sum_{k=0}^{\infty}(-1)^k\frac{x^{2k+1}}{2k+1}$ for $-1\le x\le 1$:

(a) [*3 Marks*] Compute $\displaystyle\lim_{x\to 0}\frac{\tan^{-1}(x^2) - x^2}{x^6}$.

(b) [*6 Marks*] Use the given Taylor series, to show that:

$$\int_0^1 \frac{\tan^{-1}x}{x}\,dx = \sum_{k=0}^{\infty}(-1)^k\frac{1}{(2k+1)^2}\ .$$

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9. (a) [*4 Marks*] Name the graphs of the following polar equations as *circles*, *lines*, *spirals*, *limacons* or *roses*:

(i) $r = -2$ (ii) $\theta = -2$ (iii) $r = -\theta$ (iv) $r = \dfrac{2}{\sin\theta + \cos\theta}$

(b) [*4 Marks*] Sketch the graph of $r = 2 + 2\cos\theta$ and present all the details.

END OF EXAM   <u>*TOTAL MARKS = 80*</u>