Sultan Qaboos University
College of Science
Department of Mathematics and Statistics

Summer 2010
MATH2108: Calculus II

FINAL EXAM

Date: **8 August 2010** Time Allowed: **2** Hours **45** Minutes

Answer all Questions and Show all Working

1. (a) [**6 points**] Find the volume of the solid formed by revolving the region bounded by the curves $y=x+2$ and $y=x^2$ about the horizontal line $y=4$. State the method used.

(b) [**5 points**] Compute the area of the surface formed by revolving the curve $y=\sqrt{1+x}$ for $2\le x\le 3$ about the x-axis.

(c) [**4 points**] Find the points (r,θ) at which the curves $r\cos\theta=3$ and $r=4\cos\theta$ intersect.

2. [**5+5+5 points**] Evaluate the following integrals:

(a) $\int x^2\tan^{-1}x\,dx$ (b) $\int e^x\left[\tan\left(e^x\right)+\sec\left(e^x\right)\right]^2\,dx$ (c) $\int \frac{x^2+3}{(x+1)^3}\,dx$.

3. (a) [**4+4 points**] Find the limit (if it exists) in each of the following sequences:

(i) $\left\{(-1)^n\frac{n}{n+\sqrt{n}}\right\}_{n=1}^{\infty}$ (ii) $\left\{\ln(2n)-\ln(n+2)\right\}_{n=3}^{\infty}$.

(b) [**5 points**] Determine whether the sequence $\left\{\frac{(n+1)^n}{(n+1)!}\right\}_{n=1}^{\infty}$ is monotonic.

4. (a) [**4+4 points**] Determine whether the following series converges or diverges, and state the convergence test used:

(i) $\sum_{k=1}^{\infty}\left(\frac{2\sqrt{k}+1}{2+\sqrt{k}}\right)^{k/2}$ (ii) $1+\frac{2!}{1\cdot 3}+\frac{3!}{1\cdot 3\cdot 5}+\frac{4!}{1\cdot 3\cdot 5\cdot 7}+\frac{5!}{1\cdot 3\cdot 5\cdot 7\cdot 9}+\cdots$.

(b) [**6 points**] Determine whether the series $\sum_{k=2}^{\infty}\frac{(-1)^k}{\ln k}$ converges absolutely, converges conditionally or diverges. State the convergence test used.

5. (a) [**5 points**] Find the sum of the convergent series $\sum_{k=1}^{\infty}\frac{3}{(2k-1)(2k+1)}$.

(b) [**6 points**] Find the radius and interval of convergence for the power series $\sum_{k=1}^{\infty}\frac{(x-1)^k}{k\,2^k}$.

6. (a) [**5 points**] Construct the Taylor series for $f(x) = e^{-x+1}$ at $c = 1$, answer in sigma notation.

(b) [**4 points**] Use the Taylor series in part (a) to show that $\sum_{k=0}^{\infty} \frac{(-1)^k}{k!} = \frac{1}{e}$ and $\sum_{k=0}^{\infty} \frac{1}{k!} = e$.

7. Given the series $\ln(1+x) = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{x^k}{k}$ for $-1 < x \le 1$.

(a) [**4 points**] Find the series representation for $\ln\left(\frac{1+x}{1-x}\right)$ in sigma notation.

(b) [**4 points**] Use the first three nonzero terms of the power series representation of $\ln(1+x^2)$ to evaluate the definite integral $\int_0^1 \ln(1+x^2)\,dx$.

8. [**2 points each**] **Circle** the correct answer for the following Multiple Choice Questions:

i. If the partial sum $\sum_{k=0}^{n} a_k = \pi - \frac{2}{\sqrt[3]{n}}$, then the limit $\lim_{k\to\infty} a_k$ is equal to

(A) does not exist (B) 0 (C) π (D) ∞.

ii. If the sequence $\{a_n\}_{n=1}^{\infty}$ converges to $L \ne 0$, then the alternating series $\sum_{k=1}^{\infty} (-1)^{k+1} a_k$

(A) conditionally converges (B) diverges (C) absolutely converges

(D) has the sum $L \ne 0$.

iii. If $\int_3^{\infty} f(x)\,dx$ converges to $L \ne 0$ and $a_k = f(k)$ for $k \ge 3$, then the series $\sum_{k=3}^{\infty} a_k$

(A) converges by the Integral test (B) diverges (C) has the sum $L \ne 0$

(D) may converge or diverge.

iv. One of the following series is absolutely convergent

(A) $\sum_{k=1}^{\infty} \frac{\cos \pi k}{k}$ (B) $\sum_{k=1}^{\infty} \frac{\sin \pi k}{k}$ (C) $\sum_{k=1}^{\infty} (-e)^k$ (D) $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\sqrt{k}}$.

v. If $\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = 1$, then the series $\sum_{k=1}^{\infty} a_k$

(A) converges by the Ratio test (B) absolutely converges by the Ratio test

(C) diverges by the kth-Term test (D) may diverge or converge.

9. [**1 point each**] Name the graphs of the following polar equations as *Line, Circle, Cardioid, Roses, Limacon* or *Spiral*:

(i) $r = \ln(2+\theta)$ **(ii)** $r = \frac{1}{2\sin\theta + \cos\theta}$ **(iii)** $r = 3\sin(\theta + \pi)$ **(iv)** $-\sqrt{2} = \tan\theta$ **(v)** $r = 3\sin\theta - 2\cos\theta$.

Full Mark : 100 points