Sultan Qaboos University - College of Science
MATH 2107 - Calculus I - Summer 2011

Home Work

- Please do **NOT** submit the solution of the home work.
- A quiz will be given in week 7 on Sunday based on this home work.

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1. Find the limits

(a) $\lim\limits_{x\to -4} \dfrac{|x+5|-1}{x^2-16}$ (b) $\lim\limits_{x\to 1} \dfrac{x-\sqrt{x}}{1-\sqrt{x}}$ (c) $\lim\limits_{t\to 0} \left(\dfrac{1}{t\sqrt{t+1}} - \dfrac{1}{t}\right)$

(d) $\lim\limits_{x\to \frac{\pi}{2}} \dfrac{\cos x}{x-\frac{\pi}{2}}$ (e) $\lim\limits_{x\to 0} \dfrac{\sin(3x^3)}{\sin^3(3x)}$ (f) $\lim\limits_{\theta\to 0} \dfrac{\cos\theta}{\theta}$

(g) $\lim\limits_{x\to 1} \exp\left(\dfrac{\pi-\pi^x}{\cos\left(\frac{\pi x}{2}\right)}\right)$ (h) $\lim\limits_{x\to -\infty} \sqrt{x^2-x}+x$ (i) $\lim\limits_{x\to +\infty} \dfrac{\ln(\ln x)}{\sqrt{x}}$

(j) $\lim\limits_{x\to 0^+} x^{(1-e^x)}$ (k) $\lim\limits_{x\to -1^-} \dfrac{f(x)+1}{1+x}$, where $-(2x+3) \le f(x) \le x^2-2$ for all x

(l) $\lim\limits_{h\to 0} \dfrac{\cos\sqrt{x+h}-\cos\sqrt{x}}{h}$ (m) $\lim\limits_{h\to 0} \dfrac{f(1+h)-1}{h}$, where $f(x)=x^{50}$

(n) $\lim\limits_{n\to +\infty} \left(\dfrac{-2-3\ldots-(n+2)}{(n+2)^2}\right)$.

2. Determine the value(s) of the constant k that will make the function f given by

$$f(x) = \begin{cases} k^2x^2-2x-1 & \text{if} \quad x \le 2 \\ 4 & \text{if} \quad x = 2 \\ x^2+kx+3 & \text{if} \quad x > 2 \end{cases}$$

continuous at $x=2$.

3. Let f be a function defined by

$$f(x) = \begin{cases} \dfrac{4-x^2}{x^2+5x+6} & \text{if} \quad x \le -1 \\ \ln(1-x^2) & \text{if} \quad -1 < x \le 0 \\ x^2\sin\left(\dfrac{\pi}{x^2}\right) & \text{if} \quad x > 0 \end{cases}$$

(a) Determine the interval(s) on which f is continuous.
(b) Find all vertical and horizontal asymptotes of f.

4. Use the formal definition of limit to show that $\lim\limits_{x\to 3} \dfrac{x^2+x-12}{3-x} = -7$.

5. Use the limit definition of the derivative to show that

(a) $\dfrac{d}{dx}(\tan x) = \sec^2 x$,

(b) the function f given by $f(x) = \dfrac{|x+1|-2}{1-x}$ is not differentiable at $x=-1$.

6. Let f be a function given by $f(x) = x + \dfrac{x^2 - 1}{|x| - 1}$.

(a) Determine all values of x at which f is discontinuous.

(b) Find $\lim_{x \to -1} f(x)$ and $\lim_{x \to 1} f(x)$.

(c) Sketch the graph of f.

(d) From the graph in part (c), find all values of x at which f is not differentiable.

7. Evaluate the following expressions given that

$$f(1) = 1, \quad g(1) = -1, \quad g(2) = g'(2) = \sqrt{3}, \quad f'(1) = 2, \quad g'(1) = -3,$$

(a) $\dfrac{d}{dx}\left[\dfrac{[f(x)]^2}{g(x^2)}\right]\Bigg|_{x=1}$ (b) $\displaystyle\int_1^2 \frac{g(x)g'(x)}{\sqrt{1 + [g(x)]^2}}dx$ (c) $h'(1)$, where $h(x) = f\left(\frac{1}{\sqrt{3}} g\left(2f(x)\right)\right)$.

8. Find $\dfrac{dy}{dx}$:

(a) $y = \sec^2(xe^x)$ (b) $y = \ln\left(\dfrac{x^x\sqrt{x}\tan^{-1}x}{1 + e^{\sqrt{x}}}\right)$ (c) $y = x\cos(\pi y) + \ln(xy)$

(d) $y = (\sec x \tan x)^{\cos x}$ (e) $y = x^x + x^\pi + \pi^x + \pi^\pi$

(f) $y = \displaystyle\int_2^x \sin\left(t^2 - 2\right)dt$ (g) $y = \displaystyle\int_{2x}^{x^2} \sqrt{t^2 + 1}dt$.

9. Find an equation of the tangent line to the given curve at the given point:

(a) $y = \dfrac{h(x)}{2\sqrt{x}}$ at $x = 1$, where $h(1) = h'(1) = 2$ (b) $y = \displaystyle\int_1^{\tan x} \frac{t^2}{1 + t^2}dt \left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$ at $x = \dfrac{\pi}{4}$

(c) $\sin(x + y) = \tan(xy)$ at the point $(0, \pi)$ (d) $y = f^{-1}(x)$ at $x = -1$, where $f(x) = x^3 + 2x^2 + x + 1$.

10. Let the function f be defined by

$$f(x) = \begin{cases} |x| + (x-1)^{1/3} & \text{if} \quad x < 2 \\ \dfrac{4}{x+2} + 2\cos(\pi x) & \text{if} \quad x \geq 2 \end{cases}$$

Find all points (if any) where f is: (a) discontinuous, (b) non-differentiable.

11. Let f be a function defined by

$$f(x) = \begin{cases} 1 - Ax^2 & \text{if} \quad x \leq B \\ 2 - x & \text{if} \quad x > B \end{cases}$$

Find the values of the constants A and B, so that f is differentiable at $x = B$.

12. Let $f(x) = \sqrt[3]{x}$, $x \in [0, 1]$.

(a) Show that the hypotheses of the Mean Value theorem are satisfied.

(b) Find all values of c guaranteed in the conclusion of the Mean Value Theorem.

13. Use the Mean Value theorem to show that

(a) $|\cos x - \cos y| \leq |x - y|$ for all real values of x and y.

(b) $|e^x - e^y| > |x - y|$ for all x and y in the interval $(0, +\infty)$.

14. Use an appropriate local linear approximation to estimate the value of

(a) $\sqrt[3]{7.75}$ (b) $x \sin x$ at $x = 92°$.

15. Let $f(x) = x^6 - x^4 + 3x^3 - 2x$.

(a) Use the Intermediate Value theorem to show that f has a critical number in the interval $[0, 1]$.

(b) Use Newton's method to estimate this critical number accurate to 4 decimal places.

16. Find the critical number(s) and absolute extrema of

(a) $f(x) = 2x - \sqrt{x}$ on $[0, 1]$ (b) $f(x) = \begin{cases} -4x & \text{if } -\frac{3}{2} \leq x < 0 \\ 1 - (x-1)^2 & \text{if } 0 \leq x \leq \frac{3}{2} \end{cases}$

17. Let (a) $f(x) = (5 - x)x^{2/3}$ (b) $f(x) = \dfrac{x^2}{x+1}$.

(i) Find the intercept(s) and asymptote(s) of f (if any).

(ii) Find the critical number(s) of f, the interval(s) on which f is increasing, the interval(s) on which f is decreasing and relative extrema of f (if any).

(iii) Find the interval(s) on which f is concave up, the interval(s) on which f is concave down and inflection point(s) of f (if any).

(iv) Sketch the graph of f.

18. Sketch the graph of the following functions, showing all significant features:

(a) $f(x) = \dfrac{x}{x^2+1}$ (b) $f(x) = \ln\left|x^2 - 1\right|$.

19. (a) Find the point on the graph of $y = 16x^{-4}$, $x > 0$, closest to the origin. Can we find a point which is farthest away from the origin? Explain your answer.

(b) A rectangle is to be inscribed in a right triangle enclosed between the line $2x + 3y = 6$ and the coordinate axes. Find the dimensions of the rectangle of largest area.

20. A spherical balloon is inflated so that its volume is increasing at the rate of $3ft^3/min$. How fast is the diameter of the balloon increasing when the radius is $1ft$.

21. Evaluate the integrals:

(a) $\displaystyle\int e^x(e^x+1)^2 dx$ (b) $\displaystyle\int (x - e^{4x})dx$ (c) $\displaystyle\int x\sqrt{x^2+4}\, dx$ (d) $\displaystyle\int 6x^2 \cos x^3 dx$

(e) $\displaystyle\int 4x \sec x^2 \tan x^2 dx$ (f) $\displaystyle\int \frac{(\ln x)^{10}}{x} dx$ (g) $\displaystyle\int \tan^9 2x \ \sec^2 2x \ dx$

(h) $\displaystyle\int \sin^5 3x \ \cos^2 3x \ dx$ (i) $\displaystyle\int \ln(10e^{5x})dx$ (j) $\displaystyle\int \frac{\sin \pi x}{\sqrt{\cos \pi x}} dx$ (k) $\displaystyle\int \frac{x^3 - 1}{\sqrt{x+1}} dx$.

22. Find a function f satisfying $f'(x) = 3x^2 + 1$ and $f(0) = 2$.

23. Write out all terms and compute

$$\sum_{n=1}^{6}(n^2+3n).$$

24. Writing into summation notation, compute the sum of the squares of the first 12 positive integers.

25. Compute the sum

$$\frac{1}{n^3}\sum_{m=1}^{n}\left(m^2-m\right)$$

and then the limit of the sum as $n \to \infty$.

26. Compute the sum

$$\sum_{m=3}^{n+2}\left(1+\frac{m-2}{n}\right)\frac{2}{n}.$$

27. Find the right end point approximation to the area under the curve $y = 3x^2 + 2$ over the interval $[0, 3]$, using $n = 3$ subintervals of equal length.

28. Find the area under the curve $y = x^2$ over the interval $[0, 2]$ by calculating the limit of the left end point approximation.

29. For $f(x) = x^2 - 2x$ on the interval $[0, 2]$, list the evaluation points for the Midpoint Rule with $n = 4$. Sketch the function with approximating rectangles and then evaluate the Riemann sum.

30. Find the value of the integral:

(a) $\displaystyle\int_0^5 \left|x^2-4x+3\right| dx$ (b) $\displaystyle\int_{\ln 1}^{\ln 2} \frac{e^{2x}}{e^x+3}dx$ (c) $\displaystyle\int_0^1 \frac{dx}{\sqrt{e^{3x}}}dx$ (d) $\displaystyle\int_0^{\ln\sqrt{2}} \frac{1+\cos\left(e^{-2x}\right)}{e^{2x}}dx.$

31. Show that the function

$$f(x) = \int_0^x \frac{1}{1+t^2}dt + \int_0^{\frac{1}{x}} \frac{1}{1+t^2}dt$$

is constant on $(0, \infty)$.

32. Find the critical points of

$$f(x) = \int_2^{x^2} \frac{t-1}{1+t^2}dt \qquad (-\infty < x < \infty)$$

and discuss their nature as local maxima or minima.

33. Let f be an integrable function on $[-a, a]$, then show that:

(a) $\displaystyle\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$ if f is an even function.

(b) $\displaystyle\int_{-a}^{a} f(x)dx = 0$ if f is an odd function.

34. Evaluate $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin^{11} x\cos^4 x + x^4 \sin x)dx.$

Wish You All The Best