CHAPTER 10

Section 10-2

10-1. a) 1) The parameter of interest is the difference in fill volume, $\mu_1 - \mu_2$ (note that Δ_0=0)

2) H_0 : $\mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) H_1 : $\mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$z_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

6) Reject H_0 if $z_0 < -z_{\alpha/2} = -1.96$ or $z_0 > z_{\alpha/2} = 1.96$

7) $\bar{x}_1 = 16.015$ $\quad \bar{x}_2 = 16.005$

$\sigma_1 = 0.02$ $\quad \sigma_2 = 0.025$

$n_1 = 10$ $\quad n_2 = 10$

$$z_0 = \frac{(16.015 - 16.005)}{\sqrt{\frac{(0.02)^2}{10} + \frac{(0.025)^2}{10}}} = 0.99$$

8) since -1.96 < 0.99 < 1.96, do not reject the null hypothesis and conclude there is no evidence that the two machine fill volumes differ at $\alpha = 0.05$.

b) *P-value* $= 2(1 - \Phi(0.99)) = 2(1 - 0.8389) = 0.3222$

c) Power = $1 - \beta$, where

$$\beta = \Phi\left(z_{\alpha/2} - \frac{\Delta - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \right) - \Phi\left(-z_{\alpha/2} - \frac{\Delta - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \right)$$

$$= \Phi\left(1.96 - \frac{0.04}{\sqrt{\frac{(0.02)^2}{10} + \frac{(0.025)^2}{10}}} \right) - \Phi\left(-1.96 - \frac{0.04}{\sqrt{\frac{(0.02)^2}{10} + \frac{(0.025)^2}{10}}} \right)$$

$$= \Phi(1.96 - 3.95) - \Phi(-1.96 - 3.95) = \Phi(-1.99) - \Phi(-5.91)$$

$= 0.0233 - 0$

$= 0.0233$

Power = 1 –0.0233 = 0.9967

d) $$(\bar{x}_1 - \bar{x}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(16.015 - 16.005) - 1.96\sqrt{\frac{(0.02)^2}{10} + \frac{(0.025)^2}{10}} \le \mu_1 - \mu_2 \le (16.015 - 16.005) + 1.96\sqrt{\frac{(0.02)^2}{10} + \frac{(0.025)^2}{10}}$$

$-0.0098 \le \mu_1 - \mu_2 \le 0.0298$

With 95% confidence, we believe the true difference in the mean fill volumes is between –0.0098 and 0.0298. Since 0 is contained in this interval, we can conclude there is no significant difference between the means.

e) Assume the sample sizes are to be equal, use $\alpha = 0.05$, $\beta = 0.05$, and $\Delta = 0.04$

$$n \cong \frac{\left(z_{\alpha/2} + z_\beta\right)^2 \left(\sigma_1^2 + \sigma_2^2\right)}{\delta^2} = \frac{\left(1.96 + 1.645\right)^2 \left((0.02)^2 + (0.025)^2\right)}{(0.04)^2} = 8.35, \quad n = 9,$$

use $n_1 = n_2 = 9$

10-2. 1) The parameter of interest is the difference in breaking strengths, $\mu_1 - \mu_2$ and $\Delta_0 = 10$

2) $H_0: \mu_1 - \mu_2 = 10$

3) $H_1: \mu_1 - \mu_2 > 10$

4) $\alpha = 0.05$

5) The test statistic is

$$z_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

6) Reject H_0 if $z_0 > z_\alpha = 1.645$

7) $\bar{x}_1 = 162.5$ $\quad \bar{x}_2 = 155.0$ $\quad \delta = 10$

$\sigma_1 = 1.0$ $\quad \sigma_2 = 1.0$

$n_1 = 10$ $\quad n_2 = 12$

$$z_0 = \frac{(162.5 - 155.0) - 10}{\sqrt{\frac{(1.0)^2}{10} + \frac{(1.0)^2}{12}}} = -5.84$$

8) Since -5.84 < 1.645 do not reject the null hypothesis and conclude there is insufficient evidence to support the use of plastic 1 at $\alpha = 0.05$.

10-3 $\beta = \Phi\left(1.645 - \frac{(12-10)}{\sqrt{\frac{1}{10} + \frac{1}{12}}}\right) = \Phi(-3.03) = 0.0012$, Power = 1 – 0.0012 = 0.9988 ≈ 1

$$n = \frac{(z_{\alpha/2} + z_\beta)^2 (\sigma_1^2 + \sigma_2^2)}{(\Delta - \Delta_0)^2} = \frac{(1.645 + 1.645)^2 (1+1)}{(12-10)^2} = 5.42 \cong 6$$

Yes, the sample size is adequate

10-4. a) 1) The parameter of interest is the difference in mean burning rate, $\mu_1 - \mu_2$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$z_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

6) Reject H_0 if $z_0 < -z_{\alpha/2} = -1.96$ or $z_0 > z_{\alpha/2} = 1.96$

7) $\bar{x}_1 = 18 \quad \bar{x}_2 = 24$

$\sigma_1 = 3 \quad \sigma_2 = 3$

$n_1 = 20 \quad n_2 = 20$

$$z_0 = \frac{(18-24)}{\sqrt{\frac{(3)^2}{20} + \frac{(3)^2}{20}}} = -6.32$$

8) Since $-6.32 < -1.96$ reject the null hypothesis and conclude the mean burning rates differ significantly at $\alpha = 0.05$.

b) P-value $= 2(1 - \Phi(6.32)) = 2(1-1) = 0$

c) $$\beta = \Phi\left(z_{\alpha/2} - \frac{\Delta - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}\right) - \Phi\left(-z_{\alpha/2} - \frac{\Delta - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}\right)$$

$$= \Phi\left(1.96 - \frac{2.5}{\sqrt{\frac{(3)^2}{20} + \frac{(3)^2}{20}}}\right) - \Phi\left(-1.96 - \frac{2.5}{\sqrt{\frac{(3)^2}{20} + \frac{(3)^2}{20}}}\right)$$

$$= \Phi(1.96 - 2.64) - \Phi(-1.96 - 2.64) = \Phi(-0.68) - \Phi(-4.6)$$

$= 0.24825 - 0$

$= 0.24825$

d) $$(\bar{x}_1 - \bar{x}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \leq \mu_1 - \mu_2 \leq (\bar{x}_1 - \bar{x}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(18-24) - 1.96\sqrt{\frac{(3)^2}{20} + \frac{(3)^2}{20}} \leq \mu_1 - \mu_2 \leq (18-24) + 1.96\sqrt{\frac{(3)^2}{20} + \frac{(3)^2}{20}}$$

$$-7.86 \leq \mu_1 - \mu_2 \leq -4.14$$

We are 95% confident that the mean burning rate for solid fuel propellant 2 exceeds that of propellant 1 by between 4.14 and 7.86 cm/s.

10-5. $\bar{x}_1 = 30.87 \quad \bar{x}_2 = 30.68$

$\sigma_1 = 0.10 \quad \sigma_2 = 0.15$

$n_1 = 12 \quad n_2 = 10$

a) 90% two-sided confidence interval:

$$\left(\bar{x}_1 - \bar{x}_2\right) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le \left(\bar{x}_1 - \bar{x}_2\right) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(30.87 - 30.68) - 1.645\sqrt{\frac{(0.10)^2}{12} + \frac{(0.15)^2}{10}} \le \mu_1 - \mu_2 \le (30.87 - 30.68) + 1.645\sqrt{\frac{(0.10)^2}{12} + \frac{(0.15)^2}{10}}$$

$$0.0987 \le \mu_1 - \mu_2 \le 0.2813$$

We are 90% confident that the mean fill volume for machine 1 exceeds that of machine 2 by between 0.0987 and 0.2813 fl. oz.

b) 95% two-sided confidence interval:

$$\left(\bar{x}_1 - \bar{x}_2\right) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le \left(\bar{x}_1 - \bar{x}_2\right) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(30.87 - 30.68) - 1.96\sqrt{\frac{(0.10)^2}{12} + \frac{(0.15)^2}{10}} \le \mu_1 - \mu_2 \le (30.87 - 30.68) + 1.96\sqrt{\frac{(0.10)^2}{12} + \frac{(0.15)^2}{10}}$$

$$0.0812 \le \mu_1 - \mu_2 \le 0.299$$

We are 95% confident that the mean fill volume for machine 1 exceeds that of machine 2 by between 0.0812 and 0.299 fl. oz.

Comparison of parts a and b:

As the level of confidence increases, the interval width also increases (with all other values held constant).

c) 95% upper-sided confidence interval:

$$\mu_1 - \mu_2 \le \left(\bar{x}_1 - \bar{x}_2\right) + z_{\alpha}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$\mu_1 - \mu_2 \le (30.87 - 30.68) + 1.645\sqrt{\frac{(0.10)^2}{12} + \frac{(0.15)^2}{10}}$$

$$\mu_1 - \mu_2 \le 0.2813$$

With 95% confidence, we believe the fill volume for machine 1 exceeds the fill volume of machine 2 by no more than 0.2813 fl. oz.

10-6. a) 1) The parameter of interest is the difference in mean fill volume, $\mu_1 - \mu_2$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$z_0 = \frac{(\overline{x}_1 - \overline{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

6) Reject H_0 if $z_0 < -z_{\alpha/2} = -1.96$ or $z_0 > z_{\alpha/2} = 1.96$

7) $\overline{x}_1 = 30.87$ $\overline{x}_2 = 30.68$

$\sigma_1 = 0.10$ $\sigma_2 = 0.15$

$n_1 = 12$ $n_2 = 10$

$$z_0 = \frac{(30.87 - 30.68)}{\sqrt{\frac{(0.10)^2}{12} + \frac{(0.15)^2}{10}}} = 3.42$$

8) Since 3.42 > 1.96 reject the null hypothesis and conclude the mean fill volumes of machine 1 and machine 2 differ significantly at $\alpha = 0.05$.

b) P-value $= 2(1 - \Phi(3.42)) = 2(1 - 0.99969) = 0.00062$

c) Assume the sample sizes are to be equal, use $\alpha = 0.05$, $\beta = 0.10$, and $\Delta = 0.20$

$$n \cong \frac{\left(z_{\alpha/2} + z_\beta\right)^2\left(\sigma_1^2 + \sigma_2^2\right)}{\left(\Delta - \Delta_0\right)^2} = \frac{\left(1.96 + 1.28\right)^2\left((0.10)^2 + (0.15)^2\right)}{(-0.20)^2} = 8.53, \quad n = 9, \text{ use } n_1 = n_2 = 9$$

10-7. $\overline{x}_1 = 89.6$ $\overline{x}_2 = 92.5$

$\sigma_1^2 = 1.5$ $\sigma_2^2 = 1.2$

$n_1 = 15$ $n_2 = 20$

a) 95% confidence interval:

$$\left(\overline{x}_1 - \overline{x}_2\right) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \leq \mu_1 - \mu_2 \leq \left(\overline{x}_1 - \overline{x}_2\right) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(89.6 - 92.5) - 1.96\sqrt{\frac{1.5}{15} + \frac{1.2}{20}} \leq \mu_1 - \mu_2 \leq \left(89.6 - 92.5\right) + 1.96\sqrt{\frac{1.5}{15} + \frac{1.2}{20}}$$

$$-3.684 \leq \mu_1 - \mu_2 \leq -2.116$$

With 95% confidence, we believe the mean road octane number for formulation 2 exceeds that of formulation 1 by between 2.116 and 3.684.

b) 1) The parameter of interest is the difference in mean road octane number, $\mu_1 - \mu_2$ and $\Delta_0 = 0$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$
3) $H_1: \mu_1 - \mu_2 < 0$ or $\mu_1 < \mu_2$
4) $\alpha = 0.05$
5) The test statistic is

$$z_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

6) Reject H_0 if $z_0 < -z_\alpha = -1.645$
7) $\bar{x}_1 = 89.6$ $\bar{x}_2 = 92.5$

$\sigma_1^2 = 1.5$ $\sigma_2^2 = 1.2$

$n_1 = 15$ $n_2 = 20$

$$z_0 = \frac{(89.6 - 92.5)}{\sqrt{\frac{1.5}{15} + \frac{1.2}{20}}} = -7.25$$

8) Since $-7.25 < -1.645$ reject the null hypothesis and conclude the mean road octane number for formulation 2 exceeds that of formulation 1 using $\alpha = 0.05$.

c) P-value $\cong P(z \le -7.25) = 1 - P(z \le 7.25) = 1 - 1 \cong 0$

10-8. 99% level of confidence, $E = 4$, and $z_{0.005} = 2.575$

$$n \cong \left(\frac{z_{0.005}}{E}\right)^2 (\sigma_1^2 + \sigma_2^2) = \left(\frac{2.575}{4}\right)^2 (9 + 9) = 7.46,$$ n = 8, use $n_1 = n_2 = 8$

10-9. 95% level of confidence, E = 1, and $z_{0.025} = 1.96$

$$n \cong \left(\frac{z_{0.025}}{E}\right)^2 (\sigma_1^2 + \sigma_2^2) = \left(\frac{1.96}{1}\right)^2 (1.5 + 1.2) = 10.37,$$ n = 11, use $n_1 = n_2 = 11$

10-10.

<u>Case 1: Before Process Change</u>	<u>Case 2: After Process Change</u>
μ_1 = mean batch viscosity before change	μ_2 = mean batch viscosity after change
$\bar{x}_1 = 750.2$	$\bar{x}_2 = 756.88$
$\sigma_1 = 20$	$\sigma_2 = 20$
$n_1 = 15$	$n_2 = 8$

90% confidence on $\mu_1 - \mu_2$, the difference in mean batch viscosity before and after process change:

$$(\bar{x}_1 - \bar{x}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(750.2 - 756.88) - 1.645\sqrt{\frac{(20)^2}{15} + \frac{(20)^2}{8}} \le \mu_1 - \mu_2 \le (750.2 - 756/88) + 1.645\sqrt{\frac{(20)^2}{15} + \frac{(20)^2}{8}}$$

$$-21.08 \le \mu_1 - \mu_2 \le 7.72$$

We are 90% confident that the difference in mean batch viscosity before and after the process change lies within -21.08 and 7.72. Since 0 is contained in this interval we can conclude with 90% confidence that the mean batch viscosity was unaffected by the process change.

10-11. <u>Catalyst 1</u> <u>Catalyst 2</u>

$\bar{x}_1 = 65.22$ $\qquad$ $\bar{x}_2 = 68.42$

$\sigma_1 = 3$ $\qquad$ $\sigma_2 = 3$

$n_1 = 10$ $\qquad$ $n_2 = 10$

a) 95% confidence interval on $\mu_1 - \mu_2$, the difference in mean active concentration

$$\left(\bar{x}_1 - \bar{x}_2\right) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le \left(\bar{x}_1 - \bar{x}_2\right) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(65.22 - 68.42) - 1.96\sqrt{\frac{(3)^2}{10} + \frac{(3)^2}{10}} \le \mu_1 - \mu_2 \le \left(65.22 - 68.42\right) + 1.96\sqrt{\frac{(3)^2}{10} + \frac{(3)^2}{10}}$$

$$-5.83 \le \mu_1 - \mu_2 \le -0.57$$

We are 95% confident that the mean active concentration of catalyst 2 exceeds that of catalyst 1 by between 0.57 and 5.83 g/l.

b) Yes, since the 95% confidence interval did not contain the value 0, we would conclude that the mean active concentration depends on the choice of catalyst.

10-12. a) 1) The parameter of interest is the difference in mean batch viscosity before and after the process change, $\mu_1 - \mu_2$

2) $H_0: \mu_1 - \mu_2 = 10$

3) $H_1: \mu_1 - \mu_2 < 10$

4) $\alpha = 0.10$

5) The test statistic is

$$z_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

6) Reject H_0 if $z_0 < -z_\alpha$ where $z_{0.1} = -1.28$

7) $\bar{x}_1 = 750.2$ $\qquad$ $\bar{x}_2 = 756.88$ $\qquad$ $\Delta_0 = 10$

$\sigma_1 = 20$ $\qquad$ $\sigma_2 = 20$

$n_1 = 15$ $\qquad$ $n_2 = 8$

$$z_0 = \frac{(750.2 - 756.88) - 10}{\sqrt{\frac{(20)^2}{15} + \frac{(20)^2}{8}}} = -1.90$$

8) Since $-1.90 < -1.28$ reject the null hypothesis and conclude the process change has increased the mean by less than 10.

b) P-value = $P(z \le -1.90) = 1 - P(z \le 1.90) = 1 - 0.97128 = 0.02872$

c) Parts a and b above give evidence that the mean batch viscosity change is less than 10. This conclusion is also seen by the confidence interval given in a previous problem since the interval did not contain the value 10. Since the upper endpoint is 7.72, then this also gives evidence that the difference is less than 10.

10-13. 1) The parameter of interest is the difference in mean active concentration, $\mu_1 - \mu_2$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$
3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$
4) $\alpha = 0.05$
5) The test statistic is

$$z_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

6) Reject H_0 if $z_0 < -z_{\alpha/2} = -1.96$ or $z_0 > z_{\alpha/2} = 1.96$
7) 7) $\bar{x}_1 = 65.22$ $\bar{x}_2 = 68.42$ $\delta = 0$
$\sigma_1 = 3$ $\sigma_2 = 3$
$n_1 = 10$ $n_2 = 10$

$$z_0 = \frac{(65.22 - 68.42) - 0}{\sqrt{\frac{9}{10} + \frac{9}{10}}} = -2.385$$

8) Since $-2.385 < -1.96$ reject the null hypothesis and conclude the mean active concentrations do differ significantly at $\alpha = 0.05$.

P-value = $2(1 - \Phi(2.385)) = 2(1 - 0.99146) = 0.0171$

The conclusions reached by the confidence interval of the previous problem and the test of hypothesis conducted here are the same. A two-sided confidence interval can be thought of as representing the "acceptance region" of a hypothesis test, given that the level of significance is the same for both procedures. Thus if the value of the parameter under test that is specified in the null hypothesis falls outside the confidence interval, this is equivalent to rejecting the null hypothesis.

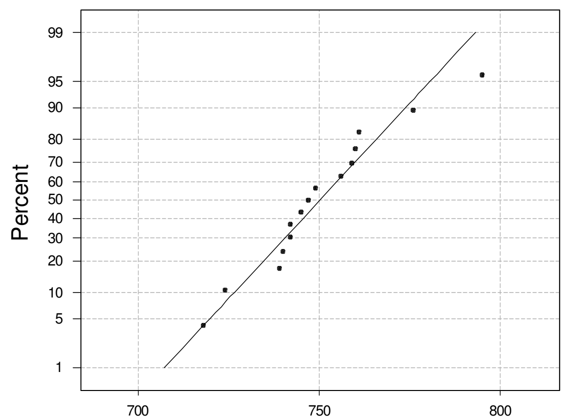
10-14.

$$\beta = \Phi\left(1.96 - \frac{(5)}{\sqrt{\frac{3^2}{10} + \frac{3^2}{10}}}\right) - \Phi\left(-1.96 - \frac{(5)}{\sqrt{\frac{3^2}{10} + \frac{3^2}{10}}}\right)$$
$$= \Phi(-1.77) - \Phi(-5.69) = 0.038364 - 0$$
$$= 0.038364$$

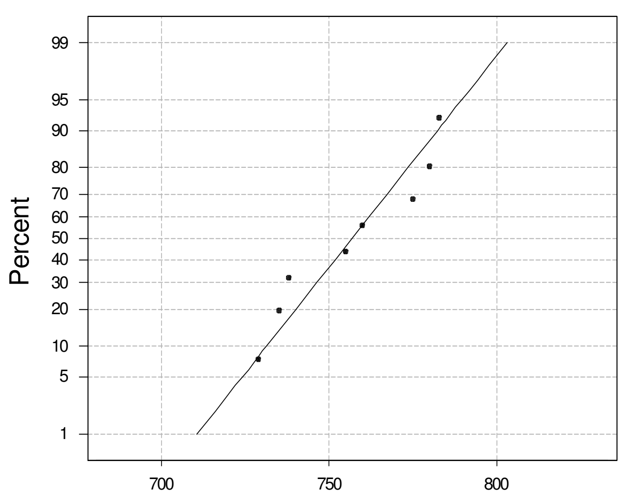
Power = $1 - \beta = 1 - 0.038364 = 09616$. it would appear that the sample sizes are adequate to detect the difference of 5, based on the power. Calculate the value of n using α and β.

$$n \cong \frac{(z_{\alpha/2} + z_\beta)^2(\sigma_1^2 + \sigma_2^2)}{(\Delta - \Delta_0)^2} = \frac{(1.96 + 1.77)^2(9+9)}{(5)^2} = 10.02,$$ Therefore, 10 is just slightly too few samples.

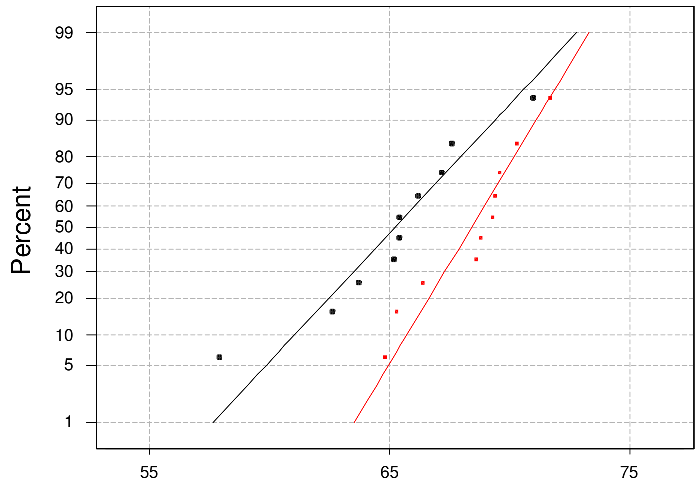
10-15 The data from the first sample n=15 appear to be normally distributed.



The data from the second sample n=8 appear to be normally distributed



10-16 The data all appear to be normally distributed based on the normal probability plot below.



Section 10-3

10-17. a) 1) The parameter of interest is the difference in mean rod diameter, $\mu_1 - \mu_2$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{(\overline{x}_1 - \overline{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

6) Reject the null hypothesis if $t_0 < -t_{\alpha/2,n_1+n_2-2}$ where $-t_{0.025,30} = -2.042$ or $t_0 > t_{\alpha/2,n_1+n_2-2}$ where $t_{0.025,30} = 2.042$

7)) $\overline{x}_1 = 8.73$ $\quad \overline{x}_2 = 8.68$

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$$

$s_1^2 = 0.35$ $\quad s_2^2 = 0.40$

$$= \sqrt{\frac{14(0.35)+16(0.40)}{30}} = 0.614$$

$n_1 = 15$ $\quad n_2 = 17$

$$t_0 = \frac{(8.73 - 8.68)}{0.614\sqrt{\frac{1}{15} + \frac{1}{17}}} = 0.230$$

8) Since $-2.042 < 0.230 < 2.042$, do not reject the null hypothesis and conclude the two machines do not produce rods with significantly different mean diameters at $\alpha = 0.05$.

b) P-value = $2P(t > 0.230) > 2(0.40)$, P-value > 0.80

c) 95% confidence interval: $t_{0.025,30} = 2.042$

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha/2,n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \le \mu_1 - \mu_2 \le \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha/2,n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

$$(8.73 - 8.68) - 2.042(0.614)\sqrt{\frac{1}{15}+\frac{1}{17}} \le \mu_1 - \mu_2 \le \left(8.73 - 8.68\right) + 2.042(0.643)\sqrt{\frac{1}{15}+\frac{1}{17}}$$

$$-0.394 \le \mu_1 - \mu_2 \le 0.494$$

Since zero is contained in this interval, we are 95% confident that machine 1 and machine 2 do not produce rods whose diameters are significantly different.

10-18. Assume the populations follow normal distributions and $\sigma_1^2 = \sigma_2^2$. The assumption of equal variances may be permitted in this case since it is known that the t-test and confidence intervals involving the t-distribution are robust to this assumption of equal variances when sample sizes are equal.

Case 1: AFCC	Case 2: ATC
μ_1 = mean foam expansion for AFCC	μ_2 = mean foam expansion for ATC
$\bar{x}_1 = 4.7$	$\bar{x}_2 = 6.9$
$s_1 = 0.6$	$s_2 = 0.8$
$n_1 = 5$	$n_2 = 5$

95% confidence interval: $t_{0.025,8} = 2.306$

$$s_p = \sqrt{\frac{4(0.60) + 4(0.80)}{8}} = 0.7071$$

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha/2,n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \le \mu_1 - \mu_2 \le \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha/2,n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

$$(4.7 - 6.9) - 2.306\,(0.7071)\sqrt{\frac{1}{5}+\frac{1}{5}} \le \mu_1 - \mu_2 \le \left(4.7 - 6.9\right) + 2.306\,(0.7071)\sqrt{\frac{1}{5}+\frac{1}{5}}$$

$$-3.23 \le \mu_1 - \mu_2 \le -1.17$$

Yes, with 95% confidence, we believe the mean foam expansion for ATC exceeds that of AFCC by between 1.17 and 3.23.

10-19. a) 1) The parameter of interest is the difference in mean catalyst yield, $\mu_1 - \mu_2$, with $\Delta_0 = 0$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 < 0$ or $\mu_1 < \mu_2$

4) $\alpha = 0.01$

5) The test statistic is

$$t_0 = \frac{(\overline{x}_1 - \overline{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

6) Reject the null hypothesis if $t_0 < -t_{\alpha, n_1+n_2-2}$ where $-t_{0.01,25} = -2.485$

7) $\overline{x}_1 = 86$ $\quad \overline{x}_2 = 89$

$s_1 = 3$ $\quad s_2 = 2$

$n_1 = 12$ $\quad n_2 = 15$

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{11(3)^2 + 14(2)^2}{25}} = 2.4899$$

$$t_0 = \frac{(86-89)}{2.4899\sqrt{\frac{1}{12} + \frac{1}{15}}} = -3.11$$

8) Since $-3.11 < -2.787$, reject the null hypothesis and conclude that the mean yield of catalyst 2 significantly exceeds that of catalyst 1 at $\alpha = 0.01$.

b) 99% confidence interval: $t_{0.005,19} = 2.861$

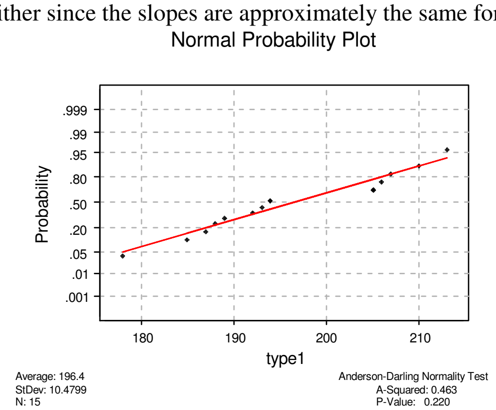
$$\left(\overline{x}_1 - \overline{x}_2\right) - t_{\alpha/2, n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \le \mu_1 - \mu_2 \le \left(\overline{x}_1 - \overline{x}_2\right) + t_{\alpha/2, n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

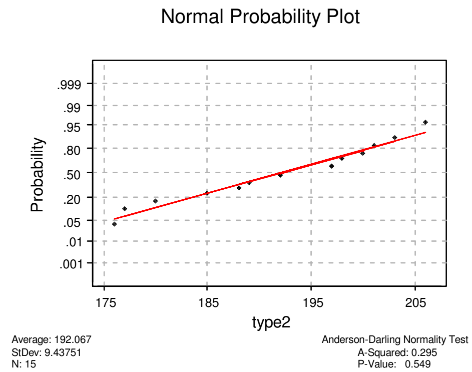
$$(86-89) - 2.787\,(2.4899)\sqrt{\frac{1}{12} + \frac{1}{15}} \le \mu_1 - \mu_2 \le (86-89) + 2.787\,(2.4899)\sqrt{\frac{1}{12} + \frac{1}{15}}$$

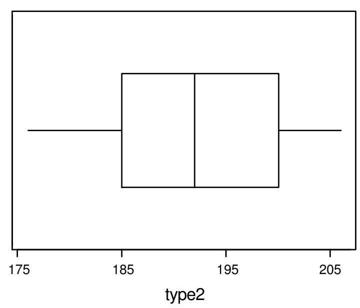
$$-5.688 \le \mu_1 - \mu_2 \le -0.3122$$

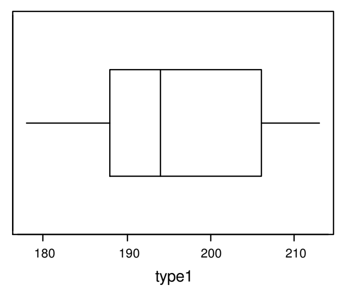
We are 95% confident that the mean yield of catalyst 2 exceeds that of catalyst 1 by between 0.3122 and 5.688

10-20. a) According to the normal probability plots, the assumption of normality appears to be met since the data fall approximately along a straight line. The equality of variances does not appear to be severely violated either since the slopes are approximately the same for both samples.









b) 1) The parameter of interest is the difference in deflection temperature under load, $\mu_1 - \mu_2$, with $\Delta_0 = 0$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 < 0$ or $\mu_1 < \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{(\overline{x}_1 - \overline{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

6) Reject the null hypothesis if $t_0 < -t_{\alpha, n_1+n_2-2}$ where $-t_{0.05,28} = -1.701$

7) Type 1   Type 2

$\bar{x}_1 = 196.4$ $\bar{x}_2 = 192.067$

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$$

$s_1 = 10.48$ $s_2 = 9.44$

$$= \sqrt{\frac{14(10.48)^2 + 14(9.44)^2}{28}} = 9.97$$

$n_1 = 15$ $n_2 = 15$

$$t_0 = \frac{(196.4 - 192.067)}{9.97\sqrt{\frac{1}{15}+\frac{1}{15}}} = 1.19$$

8) Since $1.19 > -1.701$ do not reject the null hypothesis and conclude the mean deflection temperature under load for type 2 does not significantly exceed the mean deflection temperature under load for type 1 at the 0.05 level of significance.

c) P-value = $2P(t < 1.19)$ $0.75 <$ p-value < 0.90

d) $\Delta = 5$ Use s_p as an estimate of σ:

$$d = \frac{\mu_2 - \mu_1}{2s_p} = \frac{5}{2(9.97)} = 0.251$$

Using Chart VI g) with $\beta = 0.10$, $d = 0.251$ we get $n \cong 100$. So, since $n^* = 2n-1$, $n_1 = n_2 = 51$; Therefore, the sample sizes of 15 are inadequate.

10-21. a) 1) The parameter of interest is the difference in mean etch rate, $\mu_1 - \mu_2$, with $\Delta_0 = 0$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

6) Reject the null hypothesis if $t_0 < -t_{\alpha/2, n_1+n_2-2}$ where $-t_{0.025,18} = -2.101$ or $t_0 > t_{\alpha/2, n_1+n_2-2}$ where $t_{0.025,18} = 2.101$

7) $\bar{x}_1 = 9.97$ $\bar{x}_2 = 10.4$

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$$

$s_1 = 0.422$ $s_2 = 0.231$

$$= \sqrt{\frac{9(0.422)^2 + 9(0.231)^2}{18}} = 0.340$$

$n_1 = 10$ $n_2 = 10$

$$t_0 = \frac{(9.97 - 10.4)}{0.340\sqrt{\frac{1}{10}+\frac{1}{10}}} = -2.83$$

8) Since $-2.83 < -2.101$ reject the null hypothesis and conclude the two machines mean etch rates do significantly differ at $\alpha = 0.05$.

b) P-value = $2P(t < -2.83)$ $2(0.005) <$ P-value $< 2(0.010) = 0.010 <$ P-value < 0.020

c) 95% confidence interval: $t_{0.025,18} = 2.101$

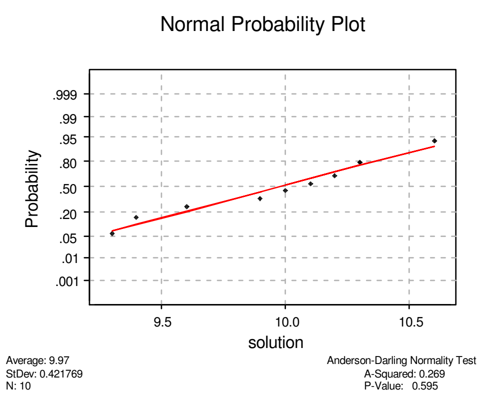
$$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2,n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \le \mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2,n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

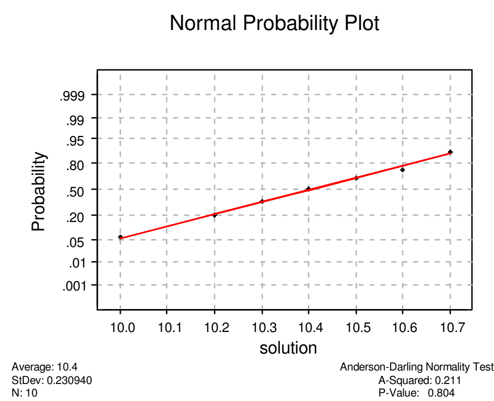
$$(9.97 - 10.4) - 2.101(.340)\sqrt{\frac{1}{10}+\frac{1}{10}} \le \mu_1 - \mu_2 \le (9.97 - 10.4) + 2.101(.340)\sqrt{\frac{1}{10}+\frac{1}{10}}$$

$$-0.7495 \le \mu_1 - \mu_2 \le -0.1105$$

We are 95% confident that the mean etch rate for solution 2 exceeds the mean etch rate for solution 1 by between 0.1105 and 0.7495.

d) According to the normal probability plots, the assumption of normality appears to be met since the data from both the samples fall approximately along a straight line. The equality of variances does not appear to be severely violated either since the slopes are approximately the same for both samples.





10-22. a) 1) The parameter of interest is the difference in mean impact strength, $\mu_1 - \mu_2$, with $\Delta_0 = 0$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 < 0$ or $\mu_1 < \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

6) Reject the null hypothesis if $t_0 < -t_{\alpha,\nu}$ where $t_{0.05,23} = 1.714$ since

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}} = 23.72$$

$\nu \cong 23$
(truncated)

7) $\bar{x}_1 = 290$ $\bar{x}_2 = 321$

$s_1 = 12$ $s_2 = 22$

$n_1 = 10$ $n_2 = 16$

$$t_0 = \frac{(290-321)}{\sqrt{\frac{(12)^2}{10}+\frac{(22)^2}{16}}} = -4.64$$

8) Since $-4.64 < -1.714$ reject the null hypothesis and conclude that supplier 2 provides gears with higher mean impact strength at the 0.05 level of significance.

b) P-value = $P(t < -4.64)$: P-value < 0.0005

c) 1) The parameter of interest is the difference in mean impact strength, $\mu_2 - \mu_1$

2) $H_0: \mu_2 - \mu_1 = 25$

3) $H_1: \mu_2 - \mu_1 > 25$ or $\mu_2 > \mu_1 + 25$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{(\bar{x}_2 - \bar{x}_1) - \delta}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}}$$

6) Reject the null hypothesis if $t_0 > t_{\alpha,\nu} = 1.708$ where

$$\nu = \frac{\left(\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1-1}+\frac{\left(\frac{s_2^2}{n_2}\right)}{n_2-1}} = 23.72$$

$$\nu \cong 23$$

7) $\bar{x}_1 = 290$ $\bar{x}_2 = 321$ $\Delta_0 = 25$ $s_1 = 12$ $s_2 = 22$ $n_1 = 10$ $n_2 = 16$

$$t_0 = \frac{(321-290)-25}{\sqrt{\frac{(12)^2}{10}+\frac{(22)^2}{16}}} = 0.898$$

8) Since $0.898 < 1.714$, do not reject the null hypothesis and conclude that the mean impact strength from supplier 2 is not at least 25 ft-lb higher that supplier 1 using $\alpha = 0.05$.

10-23. Using the information provided in Exercise 9-20, and $t_{0.025,25} = 2.06$, we find a 95% confidence interval on the difference, $\mu_2 - \mu_1$:

$$(\bar{x}_2 - \bar{x}_1) - t_{0.025,25}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}} \le \mu_2 - \mu_1 \le (\bar{x}_2 - \bar{x}_1) + t_{0.025,25}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$$

$$31 - 2.069(6.682) \le \mu_2 - \mu_1 \le 31 + 2.069(6.682)$$

$$17.175 \le \mu_2 - \mu_1 \le 44.825$$

Since the 95% confidence interval represents the differences that $\mu_2 - \mu_1$ could take on with 95% confidence, we can conclude that Supplier 2 does provide gears with a higher mean impact strength than Supplier 1. This is visible from the interval (17.175, 44.825) since zero is not contained in the interval and the differences are all positive, meaning that $\mu_2 - \mu_1 > 0$.

10-24 a) 1) The parameter of interest is the difference in mean speed, $\mu_1 - \mu_2$, $\Delta_0 = 0$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 > 0$ or $\mu_1 > \mu_2$

4) $\alpha = 0.10$

5) The test statistic is

$$t_0 = \frac{(\overline{x}_1 - \overline{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

6) Reject the null hypothesis if $t_0 > t_{\alpha, n_1+n_2-2}$ where $t_{0.10,14} = 1.345$

7) <u>Case 1: 25 mil</u> <u>Case 2: 20 mil</u>

$\overline{x}_1 = 1.15$ $\overline{x}_2 = 1.06$ $s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$

$s_1 = 0.11$ $s_2 = 0.09$ $= \sqrt{\frac{7(0.11)^2 + 7(0.09)^2}{14}} = 0.1005$

$n_1 = 8$ $n_2 = 8$

$$t_0 = \frac{(1.15 - 1.06)}{0.1005\sqrt{\frac{1}{8} + \frac{1}{8}}} = 1.79$$

8) Since 1.79 > 1.345 reject the null hypothesis and conclude reducing the film thickness from 25 mils to 20 mils significantly increases the mean speed of the film at the 0.10 level of significance (Note: since increase in film speed will result in *lower* values of observations).

b) P-value = $P(t > 1.79)$ $0.025 < \text{P-value} < 0.05$

c) 90% confidence interval: $t_{0.025,14} = 2.145$

$$(\overline{x}_1 - \overline{x}_2) - t_{\alpha/2, n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \le \mu_1 - \mu_2 \le (\overline{x}_1 - \overline{x}_2) + t_{\alpha/2, n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$(1.15 - 1.06) - 2.145(.1005)\sqrt{\frac{1}{8} + \frac{1}{8}} \le \mu_1 - \mu_2 \le (1.15 - 1.06) + 2.145(.1005)\sqrt{\frac{1}{8} + \frac{1}{8}}$$

$$-0.0178 \le \mu_1 - \mu_2 \le 0.1978$$

We are 90% confident the mean speed of the film at 20 mil exceeds the mean speed for the film at 25 mil by between -0.0178 and 0.1978 $\mu J/in^2$.

10-25. 1) The parameter of interest is the difference in mean melting point, $\mu_1 - \mu_2$, with $\Delta_0 = 0$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) $\alpha = 0.02$

5) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

6) Reject the null hypothesis if $t_0 < -t_{\alpha/2, n_1+n_2-2}$ where $-t_{0.0025,40} = -2.021$ or $t_0 > t_{\alpha/2, n_1+n_2-2}$ where $t_{0.025,40} = 2.021$

7) $\bar{x}_1 = 420$ $\quad \bar{x}_2 = 426$

$s_1 = 4$ $\quad s_2 = 3$

$n_1 = 21$ $\quad n_2 = 21$

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{20(4)^2 + 20(3)^2}{40}} = 3.536$$

$$t_0 = \frac{(420-426)}{3.536\sqrt{\frac{1}{21} + \frac{1}{21}}} = -5.498$$

8) Since $-5.498 < -2.021$ reject the null hypothesis and conclude that the data do not support the claim that both alloys have the same melting point at $\alpha = 0.02$

P-value = $2P\left(t < -5.498\right)$ P-value < 0.0010

10-26. $d = \frac{|\mu_1 - \mu_2|}{2\sigma} = \frac{3}{2(4)} = 0.375$

Using the appropriate chart in the Appendix, with $\beta = 0.10$ and $\alpha = 0.05$ we have: $n^* = 75$, so

$n = \frac{n^* + 1}{2} = 38$, $n_1 = n_2 = 38$

10-27. a) 1) The parameter of interest is the difference in mean wear amount, $\mu_1 - \mu_2$, with $\Delta_0 = 0$

.

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$
3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$
4) $\alpha = 0.05$
5) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

6) Reject the null hypothesis if $t_0 < -t_{0.025,26}$ where $-t_{0.025,26} = -2.056$ or $t_0 > t_{0.025,26}$ where $t_{0.025,26} =$ 2.056 since

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)}{n_2 - 1}} = 26.98$$

$$\nu \cong 26$$

(truncated)

7) $\bar{x}_1 = 20$ $\bar{x}_2 = 15$

$s_1 = 2$ $s_2 = 8$
$n_1 = 25$ $n_2 = 25$

$$t_0 = \frac{(20 - 15)}{\sqrt{\frac{(2)^2}{25} + \frac{(8)^2}{25}}} = 3.03$$

8) Since 3.03 > 2.056 reject the null hypothesis and conclude that the data support the claim that the two companies produce material with significantly different wear at the 0.05 level of significance.

b) P-value = 2P(t > 3.03), 2(0.0025) < P-value < 2(0.005)

0.005 < P-value < 0.010

c) 1) The parameter of interest is the difference in mean wear amount, $\mu_1 - \mu_2$

2) $H_0: \mu_1 - \mu_2 = 0$

3) $H_1: \mu_1 - \mu_2 > 0$

4) $\alpha = 0.05$

5) The test statistic is
$$t_0 = \frac{(\overline{x}_1 - \overline{x}_2) - \Delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

6) Reject the null hypothesis if $t_0 > t_{0.05,27}$ where $t_{0.05,26} = 1.706$ since

7) $\overline{x}_1 = 20$ $\quad \overline{x}_2 = 15$

$s_1 = 2$ $\quad s_2 = 8$

$n_1 = 25$ $\quad n_2 = 25$
$$t_0 = \frac{(20 - 15)}{\sqrt{\frac{(2)^2}{25} + \frac{(8)^2}{25}}} = 3.03$$

8) Since 3.03 > 1.706 reject the null hypothesis and conclude that the data support the claim that the material from company 1 has a higher mean wear than the material from company 2 using a 0.05 level of significance.

10-28 1) The parameter of interest is the difference in mean coating thickness, $\mu_1 - \mu_2$, with $\Delta_0 = 0$.

2) $H_0: \mu_1 - \mu_2 = 0$

3) $H_1: \mu_1 - \mu_2 > 0$

4) $\alpha = 0.01$

5) The test statistic is
$$t_0 = \frac{(\overline{x}_1 - \overline{x}_2) - \delta}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

6) Reject the null hypothesis if $t_0 > t_{0.01,18}$ where $t_{0.01,18} = 2.552$ since
$$v = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)}{n_2 - 1}} 18.37$$

$v \cong 18$
(truncated)

7) $\overline{x}_1 = 103.5$ $\quad \overline{x}_2 = 99.7$

$s_1 = 10.2$ $\quad s_2 = 20.1$

$n_1 = 11$ $\quad n_2 = 13$
$$t_0 = \frac{(103.5 - 99.7)}{\sqrt{\frac{(10.2)^2}{11} + \frac{(20.1)^2}{13}}} = 0.597$$

8) Since 0.597 < 2.552, do not reject the null hypothesis and conclude that increasing the temperature does not significantly reduce the mean coating thickness at $\alpha = 0.01$.

P-value = P(t > 0597), 0.25 < P-value < 0.40

10-29. If $\alpha = 0.01$, construct a 99% two-sided confidence interval on the difference to answer question 10-28.
$t_{0.005,19} = 2.878$

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha/2,\nu}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \le \mu_1 - \mu_2 \le \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha/2,\nu}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(103.5 - 99.7) - 2.878\sqrt{\frac{(10.2)^2}{11} + \frac{(20.1)^2}{13}} \le \mu_1 - \mu_2 \le (103.5 - 99.7) - 2.878\sqrt{\frac{(10.2)^2}{11} + \frac{(20.1)^2}{13}}$$

$$-14.52 \le \mu_1 - \mu_2 \le 22.12.$$

Since the interval contains 0, we are 99% confident there is no difference in the mean coating thickness between the two temperatures; that is, raising the process temperature does not significantly reduce the mean coating thickness.

10-30. 95% confidence interval:
$t_{0.025,26} = 2.056$

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha,\nu}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \le \mu_1 - \mu_2 \le \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha,\nu}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(20 - 15) - 2.056\sqrt{\frac{(2)^2}{25} + \frac{(8)^2}{25}} \le \mu_1 - \mu_2 \le (20 - 15) + 2.056\sqrt{\frac{(2)^2}{25} + \frac{(8)^2}{25}}$$

$$1.609 \le \mu_1 - \mu_2 \le 8.391$$

95% lower one-sided confidence interval:

$$t_{0.05,26} = 1.706$$

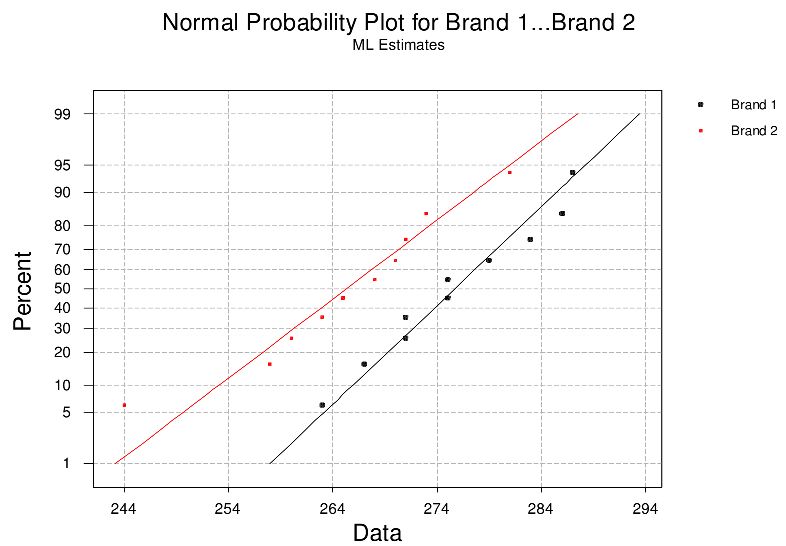
$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha,\nu}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \le \mu_1 - \mu_2$$

$$(20 - 15) - 1.706\sqrt{\frac{(2)^2}{25} + \frac{(8)^2}{25}} \le \mu_1 - \mu_2$$

$$2.186 \le \mu_1 - \mu_2$$

For part a):
We are 95% confident the mean abrasive wear from company 1 exceeds the mean abrasive wear from company 2 by between 1.609 and 8.391 mg/1000.
For part c):
We are 95% confident the mean abrasive wear from company 1 exceeds the mean abrasive wear from company 2 by at least 2.19mg/1000.

10-31 a.)



b . 1) The parameter of interest is the difference in mean overall distance, $\mu_1 - \mu_2$, with $\Delta_0 = 0$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

6) Reject the null hypothesis if $t_0 < -t_{\alpha/2, n_1+n_2-2}$ where $-t_{0.025,18} = -2.101$ or $t_0 > t_{\alpha/2, n_1+n_2-2}$ where $t_{0.025,18} = 2.101$

7) $\bar{x}_1 = 275.7$ $\bar{x}_2 = 265.3$

$s_1 = 8.03$ $s_2 = 10.04$

$n_1 = 10$ $n_2 = 10$

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

$$= \sqrt{\frac{9(8.03)^2 + 9(10.04)^2}{20}} = 9.09$$

$$t_0 = \frac{(275.7 - 265.3)}{9.09\sqrt{\frac{1}{10} + \frac{1}{10}}} = 2.558$$

8) Since 2.558>2.101 reject the null hypothesis and conclude that the data do not support the claim that both brands have the same mean overall distance at $\alpha = 0.05$. It appears that brand 1 has the higher mean differnce.

c.)P-value = $2P(t > 2.558)$ P-value $\approx 2(0.01) = 0.02$

d.) $d = \frac{5}{2(9.09)} 0.275$ β=0.95 Power =1-0.95=0.05

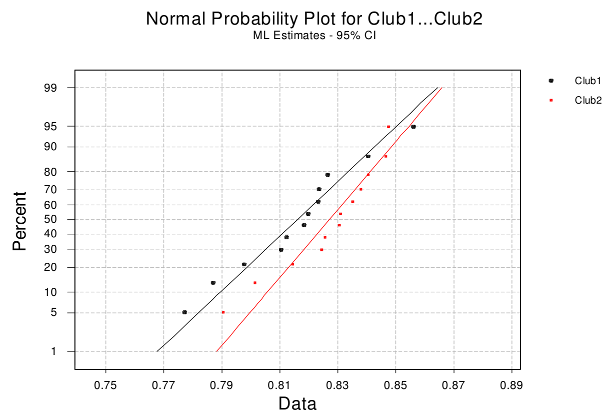
e.) 1-β=0.25 β=0..27 $d = \frac{3}{2(9.09)} = 0.165$ n*=100 $n = \frac{100 + 1}{2} = 50.5$

Therefore, n=51

f.) $\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha,v} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \le \mu_1 - \mu_2 \le \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha,v} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

$$(275.7 - 265.3) - 2.101(9.09)\sqrt{\frac{1}{10} + \frac{1}{10}} \le \mu_1 - \mu_2 \le (275.7 - 265.3) + 2.101(9.09)\sqrt{\frac{1}{10} + \frac{1}{10}}$$

$$1.86 \le \mu_1 - \mu_2 \le 18.94$$



10-32 a.)

The data appear to be normally distributed and the variances appear to be approximately equal. The slopes of the lines on the normal probability plots are almost the same.

b)

1) The parameter of interest is the difference in mean coefficient of restitution, $\mu_1 - \mu_2$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \ne 0$ or $\mu_1 \ne \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

6) Reject the null hypothesis if $t_0 < -t_{\alpha/2, n_1+n_2-2}$ where $-t_{0.025,22} = -2.074$ or $t_0 > t_{\alpha/2, n_1+n_2-2}$ where

$t_{0.025,22} = 2.074$

7) $\overline{x}_1 = 0.8161$ $\overline{x}_2 = 0.8271$

$s_1 = 0.0217$ $s_2 = 0.0175$

$n_1 = 12$ $n_2 = 12$

$$s_p = \sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{11(0.0217)^2+11(0.0175)^2}{22}} = 0.01971$$

$$t_0 = \frac{(0.8161 - 0.8271)}{0.01971\sqrt{\frac{1}{12}+\frac{1}{12}}} = -1.367$$

8) Since –1.367 > -2.074 do not reject the null hypothesis and conclude that the data do not support the claim that there is a difference in the mean coefficients of restitution for club1 and club2 at $\alpha = 0.05$

c.)P-value = $2P(t < -1.36)$ P-value ≈ 2(0.1)=0.2

d.) $d = \dfrac{0.2}{2(0.01971)} = 5.07$ $\beta \cong 0$ Power ≅1

e.) $1\text{-}\beta = 0.8$ $\beta = 0.2$ $d = \dfrac{0.1}{2(0.01971)} = 2.53$ $n^*=4$, $n = \dfrac{n^*+1}{2} = 2.5$ $n \cong 3$

f.) 95% confidence interval

$$(\bar{x}_1 - \bar{x}_2) - t_{\alpha,\nu} s_p \sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \le \mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + t_{\alpha,\nu} s_p \sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

$$(0.8161-0.8271) - 2.074(0.01971)\sqrt{\frac{1}{12}+\frac{1}{12}} \le \mu_1 - \mu_2 \le (0.8161-0.8271) + 2.074(0.01971)\sqrt{\frac{1}{12}+\frac{1}{12}}$$

$$-0.0277 \le \mu_1 - \mu_2 \le 0.0057$$

Zero is included in the confidence interval, so we would conclude that there is not a significant difference in the mean coefficient of restitution's for each club at α=0.05.

Section 10-4

10-33. $\overline{d} = 0.2736$ $s_d = 0.1356$, n = 9

95% confidence interval:

$$\overline{d} - t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right) \le \mu_d \le \overline{d} + t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right)$$

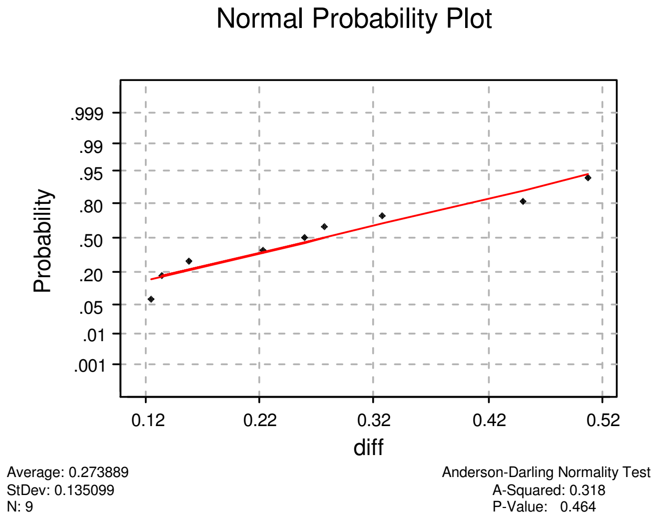
$$0.2736 - 2.306\left(\frac{0.1356}{\sqrt{9}}\right) \le \mu_d \le 0.2736 + 2.306\left(\frac{0.1356}{\sqrt{9}}\right)$$

$$0.1694 \le \mu_d \le 0.3778$$

With 95% confidence, we believe the mean shear strength of Karlsruhe method exceeds the mean shear strength of the Lehigh method by between 0.1694 and 0.3778. Since 0 is not included in this interval, the interval is consistent with rejecting the null hypothesis that the means are the same.

The 95% confidence interval is directly related to a test of hypothesis with 0.05 level of significance, and the conclusions reached are identical.

10-34. It is only necessary for the differences to be normally distributed for the paired t-test to be appropriate and reliable. Therefore, the t-test is appropriate.



10-35. 1) The parameter of interest is the difference between the mean parking times, μ_d.

2) $H_0: \mu_d = 0$

3) $H_1: \mu_d \neq 0$

4) $\alpha = 0.10$

5) The test statistic is

$$t_0 = \frac{\bar{d}}{s_d / \sqrt{n}}$$

6) Reject the null hypothesis if $t_0 < -t_{0.05,13}$ where $-t_{0.05,13} = -1.771$ or $t_0 > t_{0.05,13}$ where $t_{0.05,13} = 1.771$

7) $\bar{d} = 1.21$

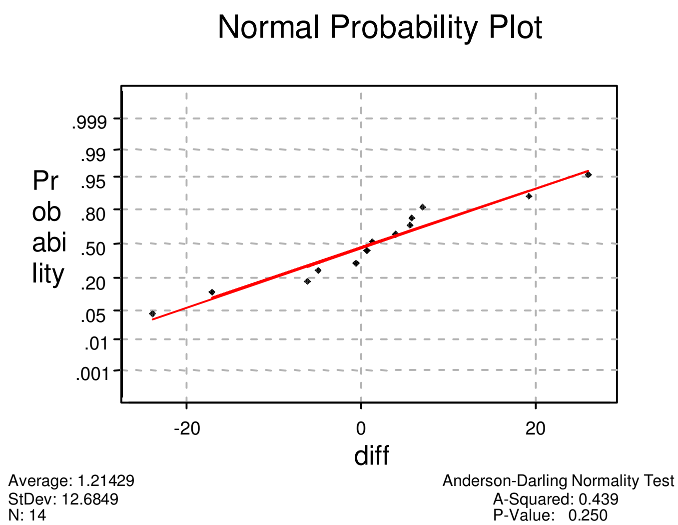
$s_d = 12.68$

$n = 14$

$$t_0 = \frac{1.21}{12.68 / \sqrt{14}} = 0.357$$

8) Since $-1.771 < 0.357 < 1.771$ do not reject the null and conclude the data do not support the claim that the two cars have different mean parking times at the 0.10 level of significance. The result is consistent with the confidence interval constructed since 0 is included in the 90% confidence interval.

10-36. According to the normal probability plots, the assumption of normality does not appear to be violated since the data fall approximately along a straight line.



10-37 $\bar{d} = 868.375$ $s_d = 1290$, n = 8 where d_i = brand 1 - brand 2

99% confidence interval:

$$\bar{d} - t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right) \le \mu_d \le \bar{d} + t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right)$$

$$868.375 - 3.499\left(\frac{1290}{\sqrt{8}}\right) \le \mu_d \le 868.375 + 3.499\left(\frac{1290}{\sqrt{8}}\right)$$

$-727.46 \le \mu_d \le 2464.21$

Since this confidence interval contains zero, we are 99% confident there is no significant difference between the two brands of tire.

10-38. a) $\bar{d} = 0.667$ $s_d = 2.964$, n = 12

95% confidence interval:

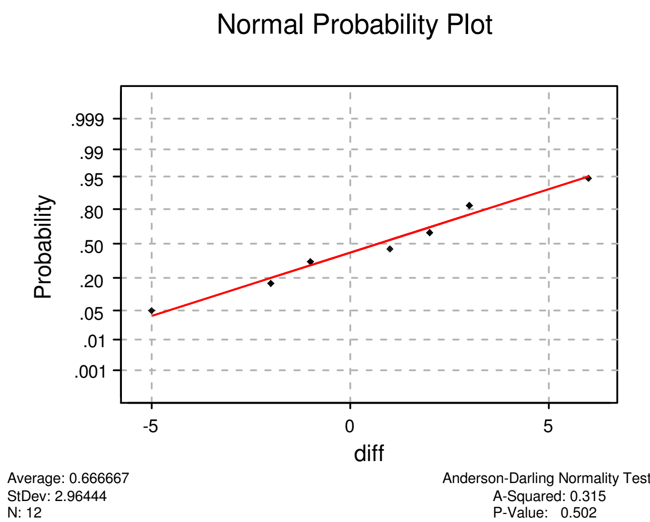
$$\bar{d} - t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right) \le \mu_d \le \bar{d} + t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right)$$

$$0.667 - 2.201\left(\frac{2.964}{\sqrt{12}}\right) \le \mu_d \le 0.667 + 2.201\left(\frac{2.964}{\sqrt{12}}\right)$$

$-1.216 \le \mu_d \le 2.55$

Since zero is contained within this interval, we are 95% confident there is no significant indication that one design language is preferable.

b) According to the normal probability plots, the assumption of normality does not appear to be violated since the data fall approximately along a straight line.



10-39. 1) The parameter of interest is the difference in blood cholesterol level, μ_d where d_i = Before − After.

2) $H_0: \mu_d = 0$

3) $H_1: \mu_d > 0$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{\bar{d}}{s_d / \sqrt{n}}$$

6) Reject the null hypothesis if $t_0 > t_{0.05,14}$ where $t_{0.05,14} = 1.761$

7) $\bar{d} = 26.867$
$s_d = 19.04$
$n = 15$

$$t_0 = \frac{26.867}{19.04/\sqrt{15}} = 5.465$$

8) Since 5.465 > 1.761 reject the null and conclude the data support the claim that the mean difference in cholesterol levels is significantly less after fat diet and aerobic exercise program at the 0.05 level of significance.

10-40. a) 1) The parameter of interest is the mean difference in natural vibration frequencies, μ_d where d_i = finite element − Equivalent Plate.

2) $H_0: \mu_d = 0$

3) $H_1: \mu_d \neq 0$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{\bar{d}}{s_d/\sqrt{n}}$$

6) Reject the null hypothesis if $t_0 < -t_{0.025,6}$ where $-t_{0.025,6} = -2.447$ or $t_0 > t_{0.005,6}$ where $t_{0.005,6}$ = 2.447

7) $\bar{d} = -5.49$
$s_d = 5.924$
$n = 7$

$$t_0 = \frac{-5.49}{5.924/\sqrt{7}} = -2.45$$

8) Since −2.447< −2.45 < 2.447, do not reject the null and conclude the data suggest that the two methods do not produce significantly different mean values for natural vibration frequency at the 0.05 level of significance.

b) 95% confidence interval:

$$\bar{d} - t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right) \le \mu_d \le \bar{d} + t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right)$$

$$-5.49 - 2.447\left(\frac{5.924}{\sqrt{7}}\right) \le \mu_d \le -5.49 + 2.447\left(\frac{5.924}{\sqrt{7}}\right)$$

$-10.969 \le \mu_d \le -0.011$

With 95% confidence, we believe that the mean difference between the natural vibration frequency from the equivalent plate method and the natural vibration frequency from the finite element method is between −10.969 and -0.011 cycles.

10-41. 1) The parameter of interest is the difference in mean weight, μ_d where d_i =Weight Before − Weight After.

2) $H_0: \mu_d = 0$

3) $H_1: \mu_d > 0$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{\bar{d}}{s_d/\sqrt{n}}$$

6) Reject the null hypothesis if $t_0 > t_{0.05,9}$ where $t_{0.05,9} = 1.833$

7) $\bar{d} = 17$
$s_d = 6.41$
$n = 10$

$$t_0 = \frac{17}{6.41/\sqrt{10}} = 8.387$$

8) Since 8.387 > 1.833 reject the null and conclude there is evidence to conclude that the mean weight loss is significantly greater than 0; that is, the data support the claim that this particular diet modification program is significantly effective in reducing weight at the 0.05 level of significance.

10-42. 1) The parameter of interest is the mean difference in impurity level, μ_d where d_i = Test 1 − Test 2.

2) $H_0: \mu_d = 0$

3) $H_1: \mu_d \neq 0$

4) $\alpha = 0.01$

5) The test statistic is

$$t_0 = \frac{\bar{d}}{s_d / \sqrt{n}}$$

6) Reject the null hypothesis if $t_0 < -t_{0.005,7}$ where $-t_{0.005,7} = -3.499$ or $t_0 > t_{0.005,7}$ where $t_{0.005,7} = 3.499$

7) $\bar{d} = -0.2125$
$s_d = 0.1727$
$n = 8$

$$t_0 = \frac{-0.2125}{0.1727/\sqrt{8}} = -3.48$$

8) Since −3.48 > -3.499 cannot reject the null and conclude the tests give significantly different impurity levels at α=0.01.

10-43. 1) The parameter of interest is the difference in mean weight loss, μ_d where d_i = Before − After.

2) $H_0: \mu_d = 10$

3) $H_1: \mu_d > 10$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{\bar{d} - \Delta_0}{s_d / \sqrt{n}}$$

6) Reject the null hypothesis if $t_0 > t_{0.05,9}$ where $t_{0.05,9} = 1.833$

7) $\bar{d} = 17$
$s_d = 6.41$
$n = 10$

$$t_0 = \frac{17 - 10}{6.41/\sqrt{10}} = 3.45$$

8) Since 3.45 > 1.833 reject the null and conclude there is evidence to support the claim that this particular diet modification program is effective in producing a mean weight loss of at least 10 lbs at the 0.05 level of significance.

10-44. Use s_d as an estimate for σ:

$$n = \left(\frac{(z_\alpha + z_\beta)\sigma_d}{10}\right)^2 = \left(\frac{(1.645 + 1.29)6.41}{10}\right)^2 = 3.53, \ n = 4$$

Yes, the sample size of 10 is adequate for this test.

Section 10-5

10-45 a) $f_{0.25,5,10} = 1.59$

d) $f_{0.75,5,10} = \dfrac{1}{f_{0.25,10,5}} = \dfrac{1}{1.89} = 0.529$

b) $f_{0.10,24,9} = 2.28$

e) $f_{0.90,24,9} = \dfrac{1}{f_{0.10,9,24}} = \dfrac{1}{1.91} = 0.525$

c) $f_{0.05,8,15} = 2.64$

f) $f_{0.95,8,15} = \dfrac{1}{f_{0.05,15,8}} = \dfrac{1}{3.22} = 0.311$

10-46 a) $f_{0.25,7,15} = 1.47$

d) $f_{0.75,7,15} = \dfrac{1}{f_{0.25,15,7}} = \dfrac{1}{1.68} = 0.596$

b) $f_{0.10,10,12} = 2.19$

e) $f_{0.90,10,12} = \dfrac{1}{f_{0.10,12,10}} = \dfrac{1}{2.28} = 0.438$

c) $f_{0.01,20,10} = 4.41$

f) $f_{0.99,20,10} = \dfrac{1}{f_{0.01,10,20}} = \dfrac{1}{3.37} = 0.297$

10-47. 1) The parameters of interest are the variances of concentration, σ_1^2, σ_2^2

2) $H_0 : \sigma_1^2 = \sigma_2^2$

3) $H_1 : \sigma_1^2 \neq \sigma_2^2$

4) $\alpha = 0.05$

5) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

6) Reject the null hypothesis if $f_0 < f_{0.975,9,15}$ where $f_{0.975,9,15} = 0.265$ or $f_0 > f_{0.025,9,15}$ where $f_{0.025,9,15} = 3.12$

7) $n_1 = 10$ $n_2 = 16$

$s_1 = 4.7$ $s_2 = 5.8$

$$f_0 = \frac{(4.7)^2}{(5.8)^2} = 0.657$$

8) Since 0.265 < 0.657 < 3.12 do not reject the null hypothesis and conclude there is insufficient evidence to indicate the two population variances differ significantly at the 0.05 level of significance.

10-48. 1) The parameters of interest are the etch-rate variances, σ_1^2, σ_2^2.

2) $H_0 : \sigma_1^2 = \sigma_2^2$

3) $H_1 : \sigma_1^2 \neq \sigma_2^2$

4) $\alpha = 0.05$

5) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

6) Reject the null hypothesis if $f_0 < f_{0.975,9,9} = 0.248$ or $f_0 > f_{0.025,9,9} = 4.03$

7) $n_1 = 10$ $\quad n_2 = 10$

$s_1 = 0.422$ $\quad s_2 = 0.231$

$$f_0 = \frac{(0.422)^2}{(0.231)^2} = 3.337$$

8) Since 0.248 < 3.337 < 4.03 do not reject the null hypothesis and conclude the etch rate variances do not differ at the 0.05 level of significance.

10-49. With $\lambda = \sqrt{2} = 1.4\ \beta = 0.10, \quad and \quad \alpha = 0.05$, we find from Chart VI o that $n_1^* = n_2^* = 100$. Therefore, the samples of size 10 would not be adequate.

10-50. a) 90% confidence interval for the ratio of variances:

$$\left(\frac{s_1^2}{s_2^2}\right) f_{1-\alpha/2, n_1-1, n_2-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{s_1^2}{s_2^2}\right) f_{\alpha/2, n_1-1, n_2-1}$$

$$\left(\frac{(0.35)}{(0.40)}\right) 0.412 \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{(0.35)}{(0.40)}\right) 2.33$$

$$0.3605 \le \frac{\sigma_1^2}{\sigma_2^2} \le 2.039$$

$$0.6004 \le \frac{\sigma_1}{\sigma_2} \le 1.428$$

b) 95% confidence interval:

$$\left(\frac{s_1^2}{s_2^2}\right) f_{1-\alpha/2, n_1-1, n_2-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{s_1^2}{s_2^2}\right) f_{\alpha/2, n_1-1, n_2-1}$$

$$\left(\frac{(0.35)}{(0.40)}\right) 0.342 \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{(0.35)}{(0.40)}\right) 2.82$$

$$0.299 \le \frac{\sigma_1^2}{\sigma_2^2} \le 2.468$$

$$0.5468 \le \frac{\sigma_1}{\sigma_2} \le 1.5710$$

The 95% confidence interval is wider than the 90% confidence interval.

c) 90% lower-sided confidence interval:

$$\left(\frac{s_1^2}{s_2^2}\right) f_{1-\alpha, n_1-1, n_2-1} \le \frac{\sigma_1^2}{\sigma_2^2}$$

$$\left(\frac{(0.35)}{(0.40)}\right) 0.500 \le \frac{\sigma_1^2}{\sigma_2^2}$$

$$0.438 \le \frac{\sigma_1^2}{\sigma_2^2}$$

$$0.661 \le \frac{\sigma_1}{\sigma_2}$$

10-51 a) 90% confidence interval for the ratio of variances:

$$\left(\frac{s_1^2}{s_2^2}\right) f_{1-\alpha/2, n_1-1, n_2-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{s_1^2}{s_2^2}\right) f_{\alpha/2, n_1-1, n_2-1}$$

$$\left(\frac{(0.6)^2}{(0.8)^2}\right) 0.156 \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{(0.6)^2}{(0.8)^2}\right) 6.39$$

$$0.08775 \le \frac{\sigma_1^2}{\sigma_2^2} \le 3.594$$

b) 95% confidence interval:

$$\left(\frac{s_1^2}{s_2^2}\right) f_{1-\alpha/2, n_1-1, n_2-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{s_1^2}{s_2^2}\right) f_{\alpha/2, n_1-1, n_2-1}$$

$$\left(\frac{(0.6)^2}{(0.8)^2}\right) 0.104 \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{(0.6)^2}{(0.8)^2}\right) 9.60$$

$$0.0585 \le \frac{\sigma_1^2}{\sigma_2^2} \le 5.4$$

The 95% confidence interval is wider than the 90% confidence interval.

c) 90% lower-sided confidence interval:

$$\left(\frac{s_1^2}{s_2^2}\right) f_{1-\alpha, n_1-1, n_2-1} \le \frac{\sigma_1^2}{\sigma_2^2}$$

$$\left(\frac{(0.6)^2}{(0.8)^2}\right) 0.243 \le \frac{\sigma_1^2}{\sigma_2^2}$$

$$0.137 \le \frac{\sigma_1^2}{\sigma_2^2}$$

10-52 1) The parameters of interest are the thickness variances, σ_1^2, σ_2^2

2) $H_0: \sigma_1^2 = \sigma_2^2$

3) $H_1: \sigma_1^2 \neq \sigma_2^2$

4) $\alpha = 0.02$

5) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

6) Reject the null hypothesis if $f_0 < f_{0.99,7,7}$ where $f_{0.99,7,7} = 0.143$ or $f_0 > f_{0.01,7,7}$ where $f_{0.01,7,7} = 6.99$

7) $n_1 = 8$ $n_2 = 8$
$s_1 = 0.11$ $s_2 = 0.09$

$$f_0 = \frac{(0.11)^2}{(0.09)^2} = 1.49$$

8) Since 0.143 < 1.49 < 6.99 do not reject the null hypothesis and conclude the thickness variances do not significantly differ at the 0.02 level of significance.

10-53 1) The parameters of interest are the strength variances, σ_1^2, σ_2^2

2) $H_0: \sigma_1^2 = \sigma_2^2$

3) $H_1: \sigma_1^2 \neq \sigma_2^2$

4) $\alpha = 0.05$

5) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

6) Reject the null hypothesis if $f_0 < f_{0.975,9,15}$ where $f_{0.975,9,15} = 0.265$ or $f_0 > f_{0.025,9,15}$ where $f_{0.025,9,15} = 3.12$

7) $n_1 = 10$ $n_2 = 16$
$s_1 = 12$ $s_2 = 22$

$$f_0 = \frac{(12)^2}{(22)^2} = 0.297$$

8) Since 0.265 < 0.297 < 3.12 do not reject the null hypothesis and conclude the population variances do not significantly differ at the 0.05 level of significance.

10-54 1) The parameters of interest are the melting variances, σ_1^2, σ_2^2

2) $H_0: \sigma_1^2 = \sigma_2^2$

3) $H_1: \sigma_1^2 \neq \sigma_2^2$

4) $\alpha = 0.05$

5) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

6) Reject the null hypothesis if $f_0 < f_{0.975,20,20}$ where $f_{0.975,20,20} = 0.4058$ or $f_0 > f_{0.025,20,20}$ where $f_{0.025,20,20} = 2.46$

7) $n_1 = 21$ $\quad$ $n_2 = 21$

$s_1 = 4$ $\quad$ $s_2 = 3$

$$f_0 = \frac{(4)^2}{(3)^2} = 1.78$$

8) Since 0.4058 < 1.78 < 2.46 do not reject the null hypothesis and conclude the population variances do not significantly differ at the 0.05 level of significance.

10-55 1) The parameters of interest are the thickness variances, σ_1^2, σ_2^2

2) $H_0: \sigma_1^2 = \sigma_2^2$

3) $H_1: \sigma_1^2 \neq \sigma_2^2$

4) $\alpha = 0.01$

5) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

6) Reject the null hypothesis if $f_0 < f_{0.995,10,12}$ where $f_{0.995,10,12}$ =0.1766 or $f_0 > f_{0.005,10,12}$ where $f_{0.005,10,12} = 5.0855$

7) $n_1 = 11$ $\quad$ $n_2 = 13$

$s_1 = 10.2$ $\quad$ $s_2 = 20.1$

$$f_0 = \frac{(10.2)^2}{(20.1)^2} = 0.2575$$

8) Since 0.1766 <0.2575 < 5.0855 do not reject the null hypothesis and conclude the thickness variances are not equal at the 0.01 level of significance.

10-56. 1) The parameters of interest are the time to assemble standard deviations, σ_1, σ_2

2) $H_0: \sigma_1^2 = \sigma_2^2$

3) $H_1: \sigma_1^2 \neq \sigma_2^2$

4) $\alpha = 0.02$

5) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

6) Reject the null hypothesis if $f_0 < f_{1-\alpha/2, n_1-1, n_2-1}$ =0..365 or $f_0 > f_{\alpha/2, n_1-1, n_2-1} = 2.86$

7) $n_1 = 25$ $\quad$ $n_2 = 21$ $\quad$ $s_1 = 0.98$ $\quad$ $s_2 = 1.02$

$$f_0 = \frac{(0.98)^2}{(1.02)^2} = 0.923$$

8) Since 0.365 < 0.923 < 2.86 do not reject the null hypothesis and conclude there is no evidence to support the claim that men and women differ significantly in repeatability for this assembly task at the 0.02 level of significance.

10-57. 98% confidence interval:

$$\left(\frac{s_1^2}{s_2^2}\right) f_{1-\alpha/2,n_1-1,n_2-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{s_1^2}{s_2^2}\right) f_{\alpha/2,n_1-1,n_2-1}$$

$$(0.923)0.365 \le \frac{\sigma_1^2}{\sigma_2^2} \le (0.923)2.86$$

$$0.3369 \le \frac{\sigma_1^2}{\sigma_2^2} \le 2.640$$

Since the value 1 is contained within this interval, we can conclude that there is no significant difference between the variance of the repeatability of men and women for the assembly task at a 98% confidence level.

10-58 For one population standard deviation being 50% larger than the other, then $\lambda = 2$. Using n =8, $\alpha = 0.01$ and Chart VI *p*, we find that $\beta \cong 0.85$. Therefore, we would say that $n = n_1 = n_2 = 8$ is not adequate to detect this difference with high probability.

10-59 1) The parameters of interest are the overall distance standard deviations, σ_1, σ_2

2) $H_0: \sigma_1^2 = \sigma_2^2$

3) $H_1: \sigma_1^2 \ne \sigma_2^2$

4) $\alpha = 0.05$

5) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

6) Reject the null hypothesis if $f_0 < f_{.0.975,9,9} = 0.248$ or $f_0 > f_{0.025,9,9} = 4.03$

7) $n_1 = 10$ $n_2 = 10$ $s_1 = 8.03$ $s_2 = 10.04$

$$f_0 = \frac{(8.03)^2}{(10.04)^2} = 0.640$$

8) Since 0.248 < 0.640 < 4.04 do not reject the null hypothesis and conclude there is no evidence to support the claim that there is a difference in the standard deviation of the overall distance of the two brands at the 0.05 level of significance.

95% confidence interval:

$$\left(\frac{s_1^2}{s_2^2}\right) f_{1-\alpha/2,n_1-1,n_2-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{s_1^2}{s_2^2}\right) f_{\alpha/2,n_1-1,n_2-1}$$

$$(0.640)0.248 \le \frac{\sigma_1^2}{\sigma_2^2} \le (0.640)4.03$$

$$0.159 \le \frac{\sigma_1^2}{\sigma_2^2} \le 2.579$$

Since the value 1 is contained within this interval, we can conclude that there is no significant difference in the variance of the distances at a 95% significance level.

10-60 1) The parameters of interest are the time to assemble standard deviations, σ_1, σ_2

2) $H_0: \sigma_1^2 = \sigma_2^2$

3) $H_1: \sigma_1^2 \neq \sigma_2^2$

4) $\alpha = 0.05$

5) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

6) Reject the null hypothesis if $f_0 < f_{.0.975,11,911} = 0.288$ or $f_0 > f_{0.025,11,11} = 3.474$

7) $n_1 = 12$ $\quad n_2 = 12$ $\quad s_1 = 0.0217$ $\quad s_2 = 0.0175$

$$f_0 = \frac{(0.0217)^2}{(0.0175)^2} = 1.538$$

8) Since 0.288<1.538<3.474 do not reject the null hypothesis and conclude there is no evidence to support the claim that there is a difference in the standard deviation of the coefficient of restitution between the two clubs at the 0.05 level of significance.

95% confidence interval:

$$\left(\frac{s_1^2}{s_2^2}\right) f_{1-\alpha/2, n_1-1, n_2-1} \leq \frac{\sigma_1^2}{\sigma_2^2} \leq \left(\frac{s_1^2}{s_2^2}\right) f_{\alpha/2, n_1-1, n_2-1}$$

$$(1.538)0.288 \leq \frac{\sigma_1^2}{\sigma_2^2} \leq (1.538)3.474$$

$$0.443 \leq \frac{\sigma_1^2}{\sigma_2^2} \leq 5.343$$

Since the value 1 is contained within this interval, we can conclude that there is no significant difference in the variances in the variances of the coefficient of restitution at a 95% significance level.

Section 10-6

10-61. 1) the parameters of interest are the proportion of defective parts, p_1 and p_2

2) $H_0: p_1 = p_2$

3) $H_1: p_1 \neq p_2$

4) $\alpha = 0.05$

5) Test statistic is

$$z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \qquad \text{where}$$

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

6) Reject the null hypothesis if $z_0 < -z_{0.025}$ where $-z_{0.025} = -1.96$ or $z_0 > z_{0.025}$ where $z_{0.025} = 1.96$

7) $n_1 = 300$ $\quad n_2 = 300$
$x_1 = 15$ $\quad x_2 = 8$

$\hat{p}_1 = 0.05$ $\hat{p}_2 = 0.0267$ $\hat{p} = \frac{15+8}{300+300} = 0.0383$

$$z_0 = \frac{0.05 - 0.0267}{\sqrt{0.0383(1-0.0383)\left(\frac{1}{300}+\frac{1}{300}\right)}} = 1.49$$

8) Since −1.96 < 1.49 < 1.96 do not reject the null hypothesis and conclude that yes the evidence indicates that there is not a significant difference in the fraction of defective parts produced by the two machines at the 0.05 level of significance.

P-value = 2(1−P(z < 1.49)) = 0.13622

10-62. 1) the parameters of interest are the proportion of satisfactory lenses, p_1 and p_2

2) $H_0 : p_1 = p_2$

3) $H_1 : p_1 \neq p_2$

4) $\alpha = 0.05$

5) Test statistic is

$$z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} \quad \text{where} \quad \hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

6) Reject the null hypothesis if $z_0 < -z_{0.005}$ where $-z_{0.005} = -258$ or $z_0 > z_{0.005}$ where $z_{0.005} = 2.58$

7) $n_1 = 300$ $n_2 = 300$
$x_1 = 253$ $x_2 = 196$

$\hat{p}_1 = 0.843$ $\hat{p}_2 = 0.653$ $\hat{p} = \frac{253+196}{300+300} = 0.748$

$$z_0 = \frac{0.843 - 0.653}{\sqrt{0.748(1-0.748)\left(\frac{1}{300}+\frac{1}{300}\right)}} = 5.36$$

8) Since 5.36 > 2.58 reject the null hypothesis and conclude that yes the evidence indicates that there is significant difference in the fraction of polishing-induced defects produced by the two polishing solutions the 0.01 level of significance.

P-value = 2(1−P(z < 5.36)) = 0

By constructing a 99% confidence interval on the difference in proportions, the same question can be answered by considering whether or not 0 is contained in the interval.

10-63. a) Power = 1 − β

$$\beta = \Phi\left(\frac{z_{\alpha/2}\sqrt{\overline{p}\,\overline{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}-(p_1-p_2)}{\hat{\sigma}_{\hat{p}_1-\hat{p}_2}}\right)-\Phi\left(\frac{-z_{\alpha/2}\sqrt{\overline{p}\,\overline{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}-(p_1-p_2)}{\hat{\sigma}_{\hat{p}_1-\hat{p}_2}}\right)$$

$$\overline{p} = \frac{300(0.05)+300(0.01)}{300+300} = 0.03 \qquad \overline{q} = 0.97$$

$$\hat{\sigma}_{\hat{p}_1-\hat{p}_2} = \sqrt{\frac{0.05(1-0.05)}{300}+\frac{0.01(1-0.01)}{300}} = 0.014$$

$$\beta = \Phi\left(\frac{1.96\sqrt{0.03(0.97)\left(\frac{1}{300}+\frac{1}{300}\right)}-(0.05-0.01)}{0.014}\right)-\Phi\left(\frac{-1.96\sqrt{0.03(0.97)\left(\frac{1}{300}+\frac{1}{300}\right)}-(0.05-0.01)}{0.014}\right)$$

$$= \Phi(-0.91)-\Phi(-4.81) = 0.18141-0 = 0.18141$$

Power = 1 − 0.18141 = 0.81859

b) $$n = \frac{\left(z_{\alpha/2}\sqrt{\frac{(p_1+p_2)(q_1+q_2)}{2}}+z_\beta\sqrt{p_1q_1+p_2q_2}\right)^2}{(p_1-p_2)^2}$$

$$= \frac{\left(1.96\sqrt{\frac{(0.05+0.01)(0.95+0.99)}{2}}+1.29\sqrt{0.05(0.95)+0.01(0.99)}\right)^2}{(0.05-0.01)^2} = 382.11$$

n = 383

10-64. a) $$\beta = \Phi\left(\frac{z_{\alpha/2}\sqrt{\overline{p}\,\overline{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}-(p_1-p_2)}{\hat{\sigma}_{\hat{p}_1-\hat{p}_2}}\right)-\Phi\left(\frac{-z_{\alpha/2}\sqrt{\overline{p}\,\overline{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}-(p_1-p_2)}{\hat{\sigma}_{\hat{p}_1-\hat{p}_2}}\right)$$

$$\overline{p} = \frac{300(0.05)+300(0.02)}{300+300} = 0.035 \qquad \overline{q} = 0.965$$

$$\hat{\sigma}_{\hat{p}_1-\hat{p}_2} = \sqrt{\frac{0.05(1-0.05)}{300}+\frac{0.02(1-0.02)}{300}} = 0.015$$

$$\beta = \Phi\left(\frac{1.96\sqrt{0.035(0.965)\left(\frac{1}{300}+\frac{1}{300}\right)}-(0.05-0.02)}{0.015}\right)-\Phi\left(\frac{-1.96\sqrt{0.035(0.965)\left(\frac{1}{300}+\frac{1}{300}\right)}-(0.05-0.02)}{0.015}\right)$$

$$= \Phi(-0.04)-\Phi(-3.96) = 0.48405-0.00004 = 0.48401$$

Power = 1 − 0.48401 = 0.51599

b) $$n = \frac{\left(z_{\alpha/2}\sqrt{\frac{(p_1+p_2)(q_1+q_2)}{2}} + z_\beta\sqrt{p_1q_1+p_2q_2} \right)^2}{(p_1-p_2)^2}$$

$$= \frac{\left(1.96\sqrt{\frac{(0.05+0.02)(0.95+0.98)}{2}} + 1.29\sqrt{0.05(0.95)+0.02(0.98)} \right)^2}{(0.05-0.02)^2} = 790.67$$

n = 791

10-65. 1) the parameters of interest are the proportion of residents in favor of an increase, p_1 and p_2

2) $H_0 : p_1 = p_2$

3) $H_1 : p_1 \neq p_2$

4) $\alpha = 0.05$

5) Test statistic is $$z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$ where $$\hat{p} = \frac{x_1+x_2}{n_1+n_2}$$

6) Reject the null hypothesis if $z_0 < -z_{0.025}$ where $-z_{0.025} = -1.96$ or $z_0 > z_{0.025}$ where $z_{0.025} = 1.96$

7) $n_1 = 500$ $\quad n_2 = 400$

$x_1 = 385$ $\quad x_2 = 267$

$\hat{p}_1 = 0.77$ $\quad \hat{p}_2 = 0.6675$ $\quad$ $$\hat{p} = \frac{385+267}{500+400} = 0.724$$

$$z_0 = \frac{0.77-0.6675}{\sqrt{0.724(1-0.724)\left(\frac{1}{500}+\frac{1}{400}\right)}} = 3.42$$

8) Since 3.42 > 1.96 reject the null hypothesis and conclude that yes the data do indicate a significant difference in the proportions of support for increasing the speed limit between residents of the two counties at the 0.05 level of significance.

P-value = 2(1−P(z < 3.42)) = 0.00062

10-66. 95% confidence interval on the difference:

$$(\hat{p}_1-\hat{p}_2) - z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1}+\frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} \le p_1-p_2 \le (\hat{p}_1-\hat{p}_2) + z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1}+\frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$(0.05-0.0267)-1.96\sqrt{\frac{0.05(1-0.05)}{300}+\frac{0.0267(1-0.0267)}{300}} \le p_1-p_2 \le (0.05-0.0267)+1.96\sqrt{\frac{0.05(1-0.05)}{300}+\frac{0.0267(1-0.0267)}{300}}$$

$-0.0074 \le p_1 - p_2 \le 0.054$

Since this interval contains the value zero, we are 95% confident there is no significant difference in the fraction of defective parts produced by the two machines and that the difference in proportions is between −0.0074 and 0.054.

10-67 95% confidence interval on the difference:

$$(\hat{p}_1-\hat{p}_2)-z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1}+\frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}\le p_1-p_2\le(\hat{p}_1-\hat{p}_2)+z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1}+\frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

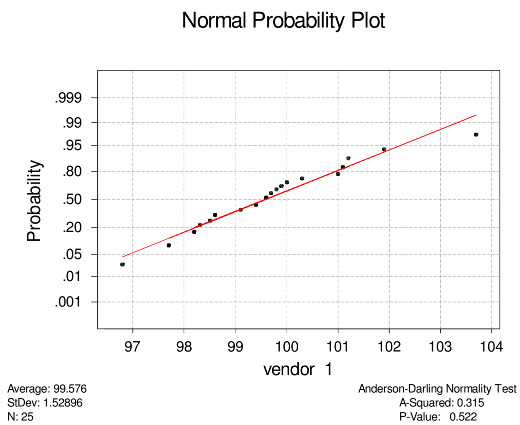
$$(0.77-0.6675)-1.96\sqrt{\frac{0.77(1-0.77)}{500}+\frac{0.6675(1-0.6675)}{400}}\le p_1-p_2\le(0.77-0.6675)+1.96\sqrt{\frac{0.77(1-0.77)}{500}+\frac{0.6675(1-0.6675)}{400}}$$

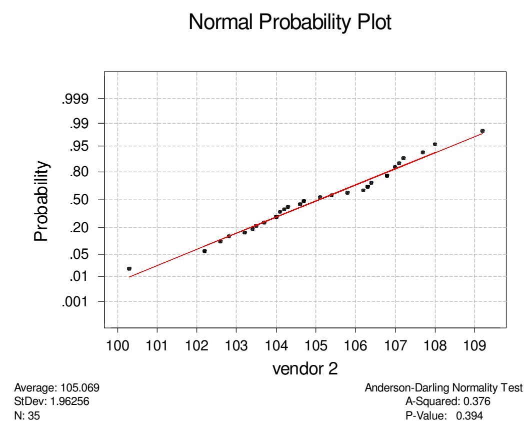
$$0.0434\le p_1-p_2\le 0.1616$$

Since this interval does not contain the value zero, we are 95% confident there is a significant difference in the proportions of support for increasing the speed limit between residents of the two counties and that the difference in proportions is between 0.0434 and 0.1616.

Supplemental Exercises

10-68 a) Assumptions that must be met are normality, equality of variance, independence of the observations and of the populations. Normality and equality of variances appears to be reasonable, see normal probability plot. The data appear to fall along a straight line and the slopes appear to be the same. Independence of the observations for each sample is assumed. It is also reasonable to assume that the two populations are independent.





b) 1) the parameters of interest are the variances of resistance of products, σ_1^2, σ_2^2

2) $H_0: \sigma_1^2 = \sigma_2^2$

3) $H_1: \sigma_1^2 \ne \sigma_2^2$

4) $\alpha = 0.05$

5) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

6) Reject H_0 if $f_0 < f_{0.975,24,34}$ where $f_{0.975,24,34} = \dfrac{1}{f_{0.025,34,24}} = \dfrac{1}{2.18} = 0.459$

or $f_0 > f_{0.025,24,34}$ where $f_{0.025,24,34} = 2.07$

7) $s_1 = 1.53$ $\quad s_2 = 1.96$
$n_1 = 25$ $\quad n_2 = 35$

$$f_0 = \frac{(1.53)^2}{(1.96)^2} = 0.609$$

8) Since 0.609 > 0.459, cannot reject H_0 and conclude the variances are significantly different at $\alpha = 0.05$.

10-69 a) Assumptions that must be met are normality, equality of variance, independence of the observations and of the populations. Normality and equality of variances appears to be reasonable, see normal probability plot. The data appear to fall along a straight line and the slopes appear to be the same. Independence of the observations for each sample is assumed. It is also reasonable to assume that the two populations are independent.

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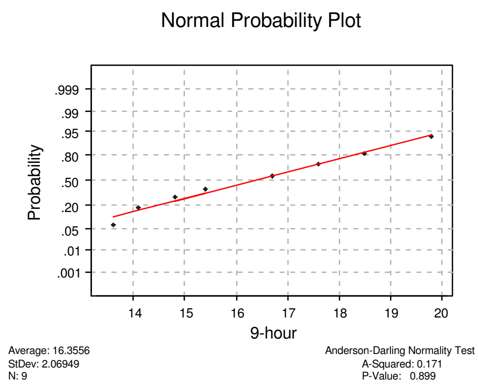
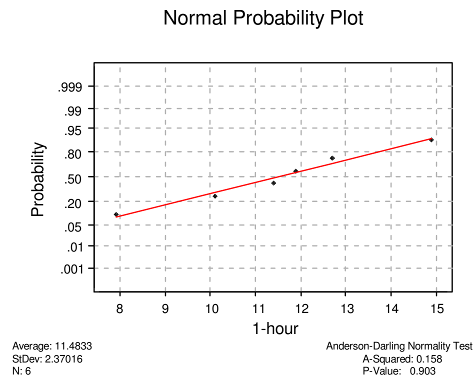




b) $\bar{x}_1 = 16.36$ $\quad \bar{x}_2 = 11.483$

$s_1 = 2.07$ $\quad s_2 = 2.37$

$n_1 = 9$ $\quad n_2 = 6$

99% confidence interval: $t_{\alpha/2,n_1+n_2-2} = t_{0.005,13}$ where $t_{0.005,13} = 3.012$

$$s_p = \sqrt{\frac{8(2.07)^2 + 5(2.37)^2}{13}} = 2.19$$

$$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2,n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \le \mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2,n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

$$(16.36 - 11.483) - 3.012(2.19)\sqrt{\frac{1}{9}+\frac{1}{6}} \le \mu_1 - \mu_2 \le (16.36 - 11.483) + 3.012(2.19)\sqrt{\frac{1}{9}+\frac{1}{6}}$$

$$1.40 \le \mu_1 - \mu_2 \le 8.36$$

c) Yes, we are 99% confident the results from the first test condition exceed the results of the second test condition by between 1.40 and 8.36 ($\times 10^6$ PA).

10-70. a) 95% confidence interval for σ_1^2 / σ_2^2

95% confidence interval on $\frac{\sigma_1^2}{\sigma_2^2}$:

$$f_{0.975,8,5} = \frac{1}{f_{0.025,5,8}} = \frac{1}{4.82} = 0.2075, \qquad f_{0.025,8,5} = 6.76$$

$$\frac{s_1^2}{s_2^2} f_{0.975,8,5} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{s_1^2}{s_2^2} f_{0.025,8,5}$$

$$\left(\frac{4.285}{5.617}\right)(0.2075) \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{4.285}{5.617}\right)(6.76)$$

$$0.1583 \le \frac{\sigma_1^2}{\sigma_2^2} \le 5.157$$

b) Since the value 1 is contained within this interval, with 95% confidence, the population variances do not differ significantly and can be assumed to be equal.

10-71 a) 1) The parameter of interest is the mean weight loss, μ_d
where d_i = Initial Weight − Final Weight.

2) $H_0 : \mu_d = 3$

3) $H_1 : \mu_d > 3$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{\bar{d} - \Delta_0}{s_d / \sqrt{n}}$$

6) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.05,7} = 1.895$.

7) $\bar{d} = 4.125$
$s_d = 1.246$
$n = 8$

$$t_0 = \frac{4.125 - 3}{1.246 / \sqrt{8}} = 2.554$$

8) Since 2.554 > 1.895, reject the null hypothesis and conclude the average weight loss is significantly greater than 3 at $\alpha = 0.05$.

b) 2) $H_0 : \mu_d = 3$

3) $H_1 : \mu_d > 3$

4) $\alpha = 0.01$

5) The test statistic is

$$t_0 = \frac{\bar{d} - \Delta_0}{s_d / \sqrt{n}}$$

6) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.01,7} = 2.998$.

7) $\bar{d} = 4.125$
$s_d = 1.246$
$n = 8$

$$t_0 = \frac{4.125 - 3}{1.246 / \sqrt{8}} = 2.554$$

8) Since 2.554 <2.998, do not reject the null hypothesis and conclude the average weight loss is not significantly greater than 3 at $\alpha = 0.01$.

c) 2) $H_0 : \mu_d = 5$

3) $H_1 : \mu_d > 5$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{\bar{d} - \Delta_0}{s_d / \sqrt{n}}$$

6) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.05,7} = 1.895$.

7) $\bar{d} = 4.125$
$s_d = 1.246$
$n = 8$

$$t_0 = \frac{4.125 - 5}{1.246 / \sqrt{8}} = -1.986$$

8) Since $-1.986 < 1.895$, do not reject the null hypothesis and conclude the average weight loss is not significantly greater than 5 at $\alpha = 0.05$.

Using $\alpha = 0.01$

2) $H_0 : \mu_d = 5$

3) $H_1 : \mu_d > 5$

4) $\alpha = 0.01$

5) The test statistic is

$$t_0 = \frac{\overline{d} - \Delta_0}{s_d / \sqrt{n}}$$

6) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.01,7} = 2.998$.

7) $\overline{d} = 4.125$

$s_d = 1.246$

$n = 8$

$$t_0 = \frac{4.125 - 5}{1.246 / \sqrt{8}} = -1.986$$

8) Since $-1.986 < 2.998$, do not reject the null hypothesis and conclude the average weight loss is not significantly greater than 5 at $\alpha = 0.01$.

10-72. $$\left(\bar{x}_1 - \bar{x}_2\right) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le \left(\bar{x}_1 - \bar{x}_2\right) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

a) 90% confidence interval: $z_{\alpha/2} = 1.65$

$$\left(88 - 91\right) - 1.65\sqrt{\frac{5^2}{20} + \frac{4^2}{20}} \le \mu_1 - \mu_2 \le \left(88 - 91\right) + 1.65\sqrt{\frac{5^2}{20} + \frac{4^2}{20}}$$

$$-5.362 \le \mu_1 - \mu_2 \le -0.638$$

Yes, with 90% confidence, the data indicate that the mean breaking strength of the yarn of manufacturer 2 exceeds that of manufacturer 1 by between 5.362 and 0.638.

b) 98% confidence interval: $z_{\alpha/2} = 2.33$

$$\left(88 - 91\right) - 2.33\sqrt{\frac{5^2}{20} + \frac{4^2}{20}} \le \mu_1 - \mu_2 \le \left(88 - 91\right) + 2.33\sqrt{\frac{5^2}{20} + \frac{4^2}{20}}$$

$$-6.340 \le \mu_1 - \mu_2 \le 0.340$$

Yes, we are 98% confident manufacturer 2 produces yarn with higher breaking strength by between 0.340 and 6.340 psi.

c) The results of parts a) and b) are different because the confidence level or z-value used is different.. Which one is used depends upon the level of confidence considered acceptable.

10-73 a) 1) The parameters of interest are the proportions of children who contract polio, p_1 , p_2

2) $H_0 : p_1 = p_2$

3) $H_1 : p_1 \neq p_2$

4) $\alpha = 0.05$

5) The test statistic is

$$z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

6) Reject H_0 if $z_0 < -z_{\alpha/2}$ or $z_0 > z_{\alpha/2}$ where $z_{\alpha/2} = 1.96$

7) $\hat{p}_1 = \frac{x_1}{n_1} = \frac{110}{201299} = 0.00055$ (Placebo) $\quad \hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = 0.000356$

$\hat{p}_2 = \frac{x_2}{n_2} = \frac{33}{200745} = 0.00016$ (Vaccine)

$$z_0 = \frac{0.00055 - 0.00016}{\sqrt{0.000356(1-0.000356)\left(\frac{1}{201299} + \frac{1}{200745}\right)}} = 6.55$$

8) Since 6.55 > 1.96 reject H_0 and conclude the proportion of children who contracted polio is significantly different at $\alpha = 0.05$.

b) $\alpha = 0.01$

Reject H_0 if $z_0 < -z_{\alpha/2}$ or $z_0 > z_{\alpha/2}$ where $z_{\alpha/2} = 2.58$

$z_0 = 6.55$

Since 6.55 > 2.58, reject H_0 and conclude the proportion of children who contracted polio is different at $\alpha = 0.01$.

c) The conclusions are the same since z_0 is so large it exceeds $z_{\alpha/2}$ in both cases.

10-74 a) $\alpha = 0.10 \qquad z_{\alpha/2} = 1.65$

$$n \cong \frac{(z_{\alpha/2})^2(\sigma_1^2 + \sigma_2^2)}{(E)^2} \cong \frac{(1.65)^2(25+16)}{(1.5)^2} = 49.61, \quad n = 50$$

b) $\alpha = 0.10 \qquad z_{\alpha/2} = 2.33$

$$n \cong \frac{(z_{\alpha/2})^2(\sigma_1^2 + \sigma_2^2)}{(E)^2} \cong \frac{(2.33)^2(25+16)}{(1.5)^2} = 98.93, \quad n = 99$$

c) As the confidence level increases, sample size will also increase.

d) $\alpha = 0.10 \qquad z_{\alpha/2} = 1.65$

$$n \cong \frac{(z_{\alpha/2})^2(\sigma_1^2 + \sigma_2^2)}{(E)^2} \cong \frac{(1.65)^2(25+16)}{(0.75)^2} = 198.44, \quad n = 199$$

e) $\alpha = 0.10 \qquad z_{\alpha/2} = 2.33$

$$n \cong \frac{(z_{\alpha/2})^2(\sigma_1^2 + \sigma_2^2)}{(E)^2} \cong \frac{(2.33)^2(25+16)}{(0.75)^2} = 395.70, \quad n = 396$$

f) As the error decreases, the required sample size increases.

10-75 $\hat{p}_1 = \frac{x_1}{n_1} = \frac{387}{1500} = 0.258$ $\hat{p}_2 = \frac{x_2}{n_2} = \frac{310}{1200} = 0.2583$

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

a) $z_{\alpha/2} = z_{0.025} = 1.96$

$$(0.258 - 0.2583) \pm 1.96\sqrt{\frac{0.258(0.742)}{1500} + \frac{0.2583(0.7417)}{1200}}$$

$$-0.0335 \le p_1 - p_2 \le 0.0329$$

Since zero is contained in this interval, we are 95% confident there is no significant difference between the proportion of unlisted numbers in the two cities.

b) $z_{\alpha/2} = z_{0.05} = 1.65$

$$(0.258 - 0.2583) \pm 1.65\sqrt{\frac{0.258(0.742)}{1500} + \frac{0.2583(0.7417)}{1200}}$$

$-0.0282 \le p_1 - p_2 \le 0.0276$

Again, the proportion of unlisted numbers in the two cities do not differ.

c) $\hat{p}_1 = \frac{x_1}{n_1} = \frac{774}{3000} = 0.258$

$\hat{p}_2 = \frac{x_2}{n_2} = \frac{620}{2400} = 0.2583$

95% confidence interval:

$$(0.258 - 0.2583) \pm 1.96\sqrt{\frac{0.258(0.742)}{3000} + \frac{0.2583(0.7417)}{2400}}$$

$-0.0238 \le p_1 - p_2 \le 0.0232$

90% confidence interval:

$$(0.258 - 0.2583) \pm 1.65\sqrt{\frac{0.258(0.742)}{3000} + \frac{0.2583(0.7417)}{2400}}$$

$$-0.0201 \le p_1 - p_2 \le 0.0195$$

Increasing the sample size decreased the error and width of the confidence intervals, but does not change the conclusions drawn. The conclusion remains that there is no significant difference.

10-76 a) 1) The parameters of interest are the proportions of those residents who wear a seat belt regularly, p_1 , p_2

2) $H_0 : p_1 = p_2$

3) $H_1 : p_1 \ne p_2$

4) $\alpha = 0.05$

5) The test statistic is

$$z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

6) Reject H_0 if $z_0 < -z_{\alpha/2}$ or $z_0 > z_{\alpha/2}$ where $z_{0.025} = 1.96$

7) $\hat{p}_1 = \frac{x_1}{n_1} = \frac{165}{200} = 0.825$ $\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = 0.807$

$\hat{p}_2 = \frac{x_2}{n_2} = \frac{198}{250} = 0.792$

$$z_0 = \frac{0.825 - 0.792}{\sqrt{0.807(1 - 0.807)\left(\frac{1}{200} + \frac{1}{250}\right)}} = 0.8814$$

8) Since $-1.96 < 0.8814 < 1.96$ do not reject H_0 and conclude that evidence is insufficient to claim that there is a difference in seat belt usage $\alpha = 0.05$.

b) $\alpha = 0.10$

Reject H_0 if $z_0 < -z_{\alpha/2}$ or $z_0 > z_{\alpha/2}$ where $z_{0.05} = 1.65$

$z_0 = 0.8814$

Since $-1.65 < 0.8814 < 1.65$, do not reject H_0 and conclude that evidence is insufficient to claim that there is a difference in seat belt usage $\alpha = 0.10$.

c) The conclusions are the same, but with different levels of confidence.

d) $n_1 = 400$, $n_2 = 500$

$\alpha = 0.05$

Reject H_0 if $z_0 < -z_{\alpha/2}$ or $z_0 > z_{\alpha/2}$ where $z_{0.025} = 1.96$

$$z_0 = \frac{0.825 - 0.792}{\sqrt{0.807(1-0.807)\left(\frac{1}{400} + \frac{1}{500}\right)}} = 1.246$$

Since $-1.96 < 1.246 < 1.96$ do not reject H_0 and conclude that evidence is insufficient to claim that there is a difference in seat belt usage $\alpha = 0.05$.

$\alpha = 0.10$

Reject H_0 if $z_0 < -z_{\alpha/2}$ or $z_0 > z_{\alpha/2}$ where $z_{0.05} = 1.65$

$z_0 = 1.012$

Since $-1.65 < 1.246 < 1.65$, do not reject H_0 and conclude that evidence is insufficient to claim that there is a difference in seat belt usage $\alpha = 0.10$.

As the sample size increased, the test statistic has also increased, since the denominator of z_0 decreased. However, the decrease (or sample size increase) was not enough to change our conclusion.

10-77. a) Yes, there could be some bias in the results due to the telephone survey.

b) If it could be shown that these populations are similar to the respondents, the results may be extended.

10-78 a) 1) The parameters of interest are the proportion of lenses that are unsatisfactory after tumble-polishing, p_1, p_2

2) $H_0 : p_1 = p_2$

3) $H_1 : p_1 \neq p_2$

4) $\alpha = 0.01$

5) The test statistic is

$$z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

6) Reject H_0 if $z_0 < -z_{\alpha/2}$ or $z_0 > z_{\alpha/2}$ where $z_{\alpha/2} = 2.58$

7) x_1 = number of defective lenses

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{47}{300} = 0.1567 \qquad \hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = 0.2517$$

$$\hat{p}_2 = \frac{x_2}{n_2} = \frac{104}{300} = 0..3467$$

$$z_0 = \frac{0.1567 - 0.3467}{\sqrt{0.2517(1-0.2517)\left(\frac{1}{300} + \frac{1}{300}\right)}} = -5.36$$

8) Since $-5.36 < -2.58$ reject H_0 and conclude there is strong evidence to support the claim that the two polishing fluids are different.

b) The conclusions are the same whether we analyze the data using the proportion unsatisfactory or proportion satisfactory. The proportion of defectives are different for the two fluids.

10-79.

$$n = \frac{\left(2.575\sqrt{\frac{(0.9+0.6)(0.1+0.4)}{2}} + 1.28\sqrt{0.9(0.1)+0.6(0.4)}\right)^2}{(0.9-0.6)^2}$$

$$= \frac{5.346}{0.09} = 59.4$$

$$n = 60$$

10-80 The parameter of interest is $\mu_1 - 2\mu_2$

$H_0: \mu_1 = 2\mu_2 \quad \rightarrow \quad H_0: \mu_1 - 2\mu_2 = 0$

$H_1: \mu_1 > 2\mu_2 \quad\quad\quad H_1: \mu_1 - 2\mu_2 > 0$

Let n_1 = size of sample 1 $\quad \overline{X}_1$ estimate for μ_1

Let n_2 = size of sample 2 $\quad \overline{X}_2$ estimate for μ_2

$\overline{X}_1 - 2\overline{X}_2$ is an estimate for $\mu_1 - 2\mu_2$

The variance is $V(\overline{X}_1 - 2\overline{X}_2) = V(\overline{X}_1) + V(2\overline{X}_2) = \frac{\sigma_1^2}{n_1} + \frac{4\sigma_2^2}{n_2}$

The test statistic for this hypothesis would then be:

$$Z_0 = \frac{(\overline{X}_1 - 2\overline{X}_2) - 0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{4\sigma_2^1}{n_2}}}$$

We would reject the null hypothesis if $z_0 > z_{\alpha/2}$ for a given level of significance.

The P-value would be $P(Z \geq z_0)$.

10-81. $H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

$n_1 = n_2 = n$

$\beta = 0.10$

$\alpha = 0.05$

Assume normal distribution and $\sigma_1^2 = \sigma_2^2 = \sigma^2$

$\mu_1 = \mu_2 + \sigma$

$$d = \frac{|\mu_1 - \mu_2|}{2\sigma} = \frac{\sigma}{2\sigma} = \frac{1}{2}$$

From Chart VI, $n^* = 50$

$$n = \frac{n^* + 1}{2} = \frac{50 + 1}{2} = 25.5$$

$n_1 = n_2 = 26$

10-82 a) $\alpha = 0.05$, $\beta = 0.05$ $\Delta = 1.5$ Use $s_p = 0.7071$ to approximate σ in equation 10-19.

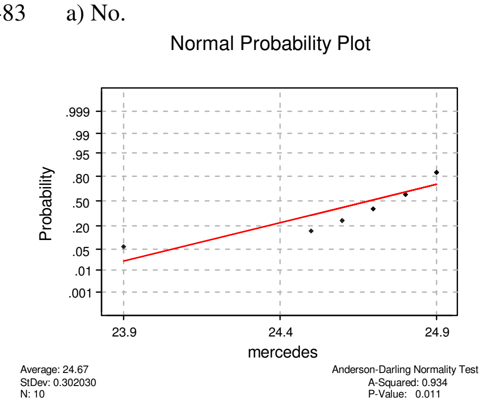
$$d = \frac{\Delta}{2(s_p)} = \frac{1.5}{2(.7071)} = 1.06 \cong 1$$

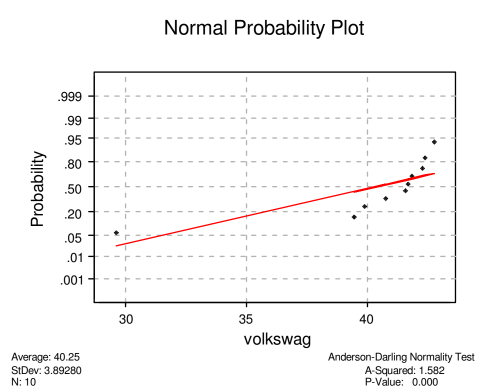
From Chart VI (e), $n^* = 20$ $n = \frac{n^* + 1}{2} = \frac{20 + 1}{2} = 10.5$

n= 11 would be needed to reject the null hypothesis that the two agents differ by 0.5 with probability of at least 0.95.

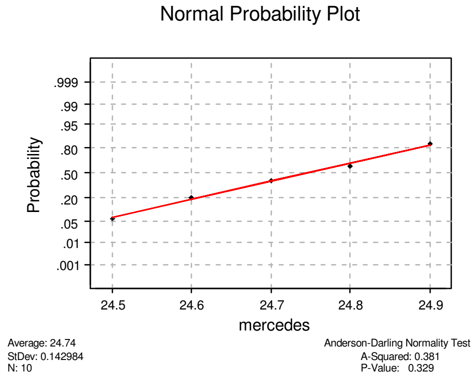
b) The original size of n = 5 in Exercise 10-18 was not appropriate to detect the difference since it is necessary for a sample size of 16 to reject the null hypothesis that the two agents differ by 1.5 with probability of at least 0.95.

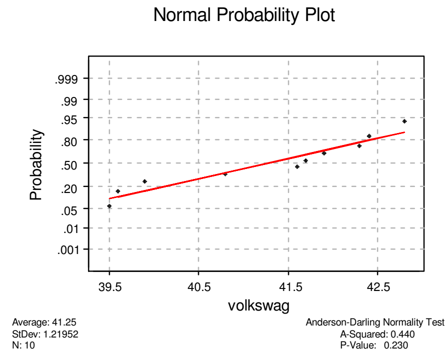
10-83 a) No.





b) The normal probability plots indicate that the data follow normal distributions since the data appear to fall along a straight line. The plots also indicate that the variances could be equal since the slopes appear to be the same.





c) By correcting the data points, it is more apparent the data follow normal distributions. Note that one unusual observation can cause an analyst to reject the normality assumption.

d) 95% confidence interval on the ratio of the variances, σ_V^2 / σ_M^2

$s_V^2 = 1.49 \qquad f_{9,9,0.025} = 4.03$

$s_M^2 = 0.0204 \qquad f_{9,9,0.975} = \dfrac{1}{f_{9,9,0.025}} = \dfrac{1}{4.03} = 0.248$

$$\left(\frac{s_V^2}{s_M^2}\right) f_{9,9,0.975} < \frac{\sigma_V^2}{\sigma_M^2} < \left(\frac{s_V^2}{s_M^2}\right) f_{9,9,0.025}$$

$$\left(\frac{1.49}{0.0204}\right) 0.248 < \frac{\sigma_V^2}{\sigma_M^2} < \left(\frac{1.49}{0.0204}\right) 4.03$$

$$18.114 < \frac{\sigma_V^2}{\sigma_M^2} < 294.35$$

Since the does not include the value of unity, we are 95% confident that there is evidence to reject the claim that the variability in mileage performance is the same for the two types of vehicles. There is evidence that the variability is greater for a Volkswagen than for a Mercedes.

10-84 1) the parameters of interest are the variances in mileage performance, σ_1^2, σ_2^2

2) $H_0 : \sigma_1^2 = \sigma_2^2$ Where Volkswagen is represented by variance 1, Mercedes by variance 2.

3) $H_1 : \sigma_1^2 \neq \sigma_2^2$

4) $\alpha = 0.05$

5) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

6) Reject H_0 if $f_0 < f_{0.975,9,9}$ where $f_{0.975,9,9} = \dfrac{1}{f_{0.025,9,9}} = \dfrac{1}{4.03} = 0.248$

or $f_0 > f_{0.025,9,9}$ where $f_{0.025,9,9} = 4.03$

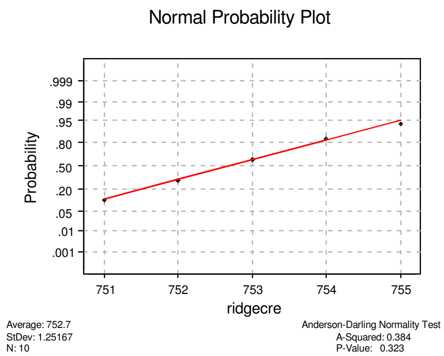
7) $s_1 = 1.22$ $\quad s_2 = 0.143$

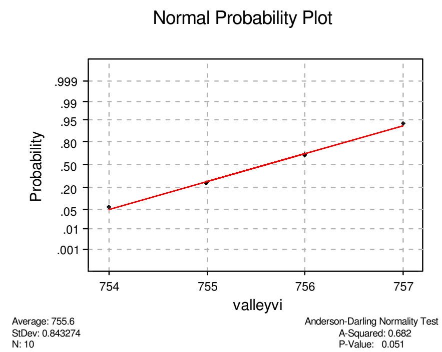
$n_1 = 10$ $\quad n_2 = 10$

$$f_0 = \frac{(1.22)^2}{(0.143)^2} = 72.78$$

8) Since 72.78 > 4.03, reject H_0 and conclude that there is a significant difference between Volkswagen and Mercedes in terms of mileage variability. Same conclusions reached in 10-83d.

10-85 a) Underlying distributions appear to be normal since the data fall along a straight line on the normal probability plots. The slopes appear to be similar, so it is reasonable to assume that $\sigma_1^2 = \sigma_2^2$.





b) 1) The parameter of interest is the difference in mean volumes, $\mu_1 - \mu_2$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \delta}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

6) Reject H_0 if $t_0 < -t_{\alpha/2,\nu}$ or $z_0 > t_{\alpha/2,\nu}$ where $t_{\alpha/2,\nu} = t_{0.025,18} = 2.101$

7) $\bar{x}_1 = 752.7$ $\quad \bar{x}_2 = 755.6$ $\qquad s_p = \sqrt{\frac{9(1.252)^2 + 9(0.843)^2}{18}} = 1.07$

$s_1 = 1.252$ $\quad s_2 = 0.843$

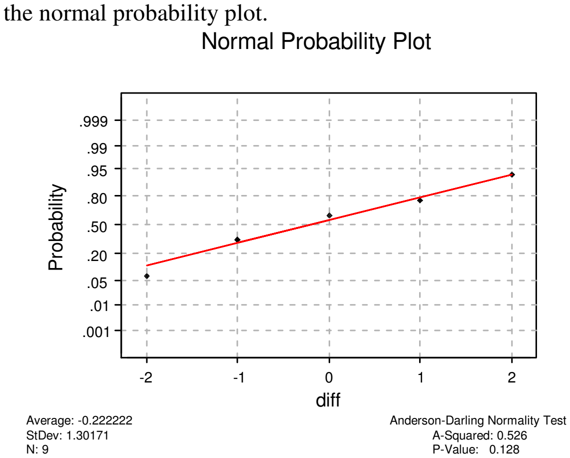
$n_1 = 10$ $\quad n_2 = 10$

$$t_0 = \frac{(752.7 - 755.6)}{1.07\sqrt{\frac{1}{10} + \frac{1}{10}}} = -6.06$$

8) Since $-6.06 < -2.101$, reject H_0 and conclude there is a significant difference between the two winery's with respect to mean fill volumes.

10-86. d=2/2(1.07)=0.93, giving a power of just under 80%. Since the power is relatively low, an increase in the sample size would increase the power of the test.

10-87. a) The assumption of normality appears to be valid. This is evident by the fact that the data lie along a straight line in the normal probability plot.



b) 1) The parameter of interest is the mean difference in tip hardness, μ_d

2) $H_0 : \mu_d = 0$

3) $H_1 : \mu_d \neq 0$

4) No significance level, calculate P-value

5) The test statistic is

$$t_0 = \frac{\bar{d}}{s_d / \sqrt{n}}$$

6) Reject H_0 if the P-value is significantly small.

7) $\bar{d} = -0.222$

$s_d = 1.30$

$n = 9$

$$t_0 = \frac{-0.222}{1.30 / \sqrt{9}} = -0.512$$

8) P-value = 2P(T < -0.512) = 2P(T > 0.512) 2(0.25) < P-value < 2(0.40)

0.50 < P-value < 0.80

Since the P-value is larger than any acceptable level of significance, do not reject H_0 and conclude there is no difference in mean tip hardness.

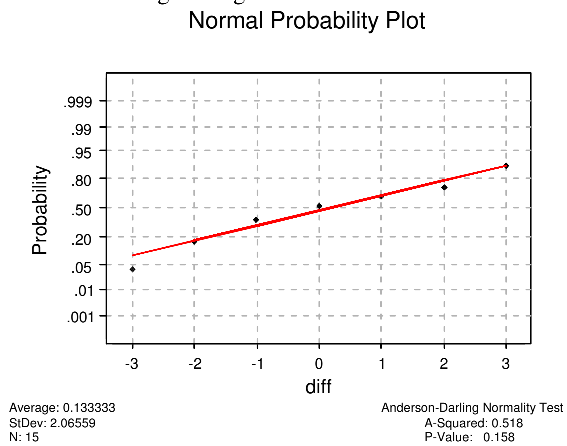
c) $\beta = 0.10$

$\mu_d = 1$

$$d = \frac{1}{\sigma_d} = \frac{1}{1.3} = 0.769$$

From Chart VI with $\alpha = 0.01$, $n = 30$

10-88. a) According to the normal probability plot the data appear to follow a normal distribution. This is evident by the fact that the data fall along a straight line.



b) 1) The parameter of interest is the mean difference in depth using the two gauges, μ_d

2) $H_0 : \mu_d = 0$

3) $H_1 : \mu_d \neq 0$

4) No significance level, calculate p-value

5) The test statistic is

$$t_0 = \frac{\bar{d}}{s_d / \sqrt{n}}$$

6) Reject H_0 if the P-value is significantly small.

7) $\bar{d} = 0.133$

$s_d = 2.065$

$n = 15$

$$t_0 = \frac{0.133}{2.065/\sqrt{15}} = 0.25$$

8) P-value = 2P(T > 0.25) $\qquad$ 2(0.40) < P-value

0.80 < P-value

Since the P-value is larger than any acceptable level of significance, do not reject H_0 and conclude there is no difference in mean depth measurements for the two gauges.

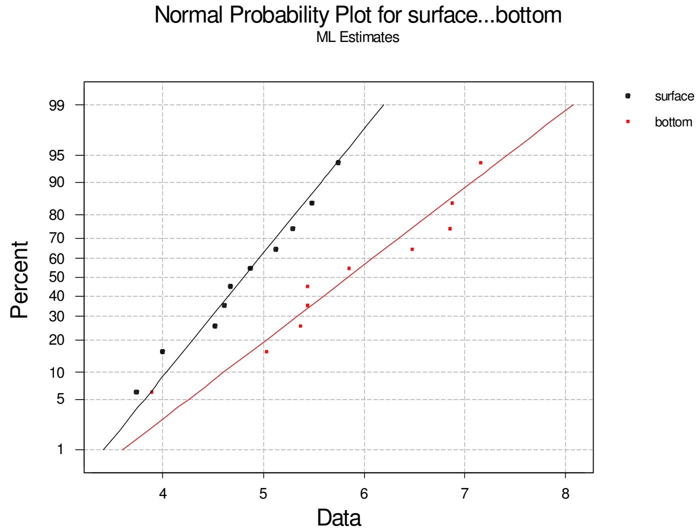
c) Power = 0.8, Therefore, since Power= 1-β , β= 0.20

$$\mu_d = 1.65$$

$$d = \frac{1.65}{\sigma_d} = \frac{1.65}{(2.065)} = 0.799$$

From Chart VI (f) with $\alpha = 0.01$ and $\beta = 0.20$, we find n =30.

10-89 a.) The data from both depths appear to be normally distributed, but the slopes are not equal. Therefore, it may not be assumed that $\sigma_1^2 = \sigma_2^2$.



b.) 1) The parameter of interest is the difference in mean HCB concentration, $\mu_1 - \mu_2$, with $\Delta_0 = 0$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

6) Reject the null hypothesis if $t_0 < -t_{0.025,15}$ where $-t_{0.025,15} = -2.131$ or $t_0 > t_{0.025,15}$ where $t_{0.025,15} =$ 2.131 since

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)}{n_2 - 1}} = 15.06$$

$\nu \cong 15$
(truncated)

7) $\bar{x}_1 = 4.804 \quad \bar{x}_2 = 5.839 \qquad s_1 = 0.631 \quad s_2 = 1.014$
$n_1 = 10 \quad n_2 = 10$

$$t_0 = \frac{(4.804 - 5.839)}{\sqrt{\frac{(0.631)^2}{10} + \frac{(1.014)^2}{10}}} = -2.74$$

8) Since –2.74 < -2.131 reject the null hypothesis and conclude that the data support the claim that the mean HCB concentration is different at the two depths sampled at the 0.05 level of significance.

b) P-value = 2P(t < -2.74), 2(0.005) < P-value < 2(0.01)

0.001 < P-value < 0.02

c) Assuming the sample sizes were equal:

a. $\Delta = 2 \quad \alpha = 0.05 \quad n_1 = n_2 = 10 \quad d = \frac{2}{2(1)} = 1$

From Chart VI (e) we find $\beta = 0.20$, and then calculate Power = 1- β = 0.80

d.)Assuming the sample sizes were equal:

$\Delta = 2 \quad \alpha = 0.05 \quad d = \frac{2}{2(1)} = 0.5$, $\beta = 0.0$

From Chart VI (e) we find n*=50 and $n = \frac{50+1}{2} = 25.5$, so n=26

Mind-Expanding Exercises

10-90 The estimate of μ is given by $\overline{X}$. Therefore, $\overline{X} = \frac{1}{2}\left(\overline{X}_1 + \overline{X}_2\right) - \overline{X}_3$. The variance of $\overline{X}$ can be shown to be: $V(\overline{X}) = \frac{1}{4}\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right) + \frac{\sigma_3^2}{n_3}$. Using s_1, s_2, and s_3 as estimates for σ_1 σ_2 and σ_3 respectively.

a) A 100(1-α)% confidence interval on μ is then:

$$\overline{X} - Z_{\alpha/2}\sqrt{\frac{1}{4}\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right) + \frac{s_3^2}{n_3}} \le \mu \le \overline{X} + Z_{\alpha/2}\sqrt{\frac{1}{4}\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right) + \frac{s_3^2}{n_3}}$$

b) A 95% confidence interval for μ is

$$\left(\frac{1}{2}(4.6+5.2)-6.1\right)-1.96\sqrt{\frac{1}{4}\left(\frac{0.7^2}{100}+\frac{0.6^2}{120}\right)+\frac{0.8^2}{130}} \le \mu \le \left(\frac{1}{2}(4.6+5.2)-6.1\right)+1.96\sqrt{\frac{1}{4}\left(\frac{0.7^2}{100}+\frac{0.6^2}{120}\right)+\frac{0.8^2}{130}}$$
$$-1.2-0.163 \le \mu \le -1.2+0.163$$
$$-1.363 \le \mu \le -1.037$$

Since zero is not contained in this interval, and because the possible differences (-1.363, -1.037) are negative, we can conclude that there is sufficient evidence to indicate that pesticide three is more effective.

10-91 The $V(\overline{X}_1 - \overline{X}_2) = \dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}$ and suppose this is to equal a constant k. Then, we are to minimize $C_1 n_1 + C_2 n_2$ subject to $\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2} = k$. Using a Lagrange multiplier, we minimize by setting the partial derivatives of $f(n_1, n_2, \lambda) = C_1 n_1 + C_2 n_2 + \lambda\left(\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2} - k\right)$ with respect to n_1, n_2 and λ equal to zero.

These equations are

$$\frac{\partial}{\partial n_1} f(n_1, n_2, \lambda) = C_1 - \frac{\lambda \sigma_1^2}{n_1^2} = 0 \qquad (1)$$

$$\frac{\partial}{\partial n_2} f(n_1, n_2, \lambda) = C_2 - \frac{\lambda \sigma_2^2}{n_2^2} = 0 \qquad (2)$$

$$\frac{\partial}{\partial \lambda} f(n_1, n_2, \lambda) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2} = k \qquad (3)$$

Upon adding equations (1) and (2), we obtain $C_1 + C_2 - \lambda\left(\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}\right) = 0$

Substituting from equation (3) enables us to solve for λ to obtain $\dfrac{C_1 + C_2}{k} = \lambda$

Then, equations (1) and (2) are solved for n_1 and n_2 to obtain

$$n_1 = \frac{\sigma_1^2 (C_1 + C_2)}{k C_1} \qquad n_2 = \frac{\sigma_2^2 (C_1 + C_2)}{k C_2}$$

It can be verified that this is a minimum and that with these choices for n_1 and n_2.

$$V(\overline{X}_1 - \overline{X}_2) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}.$$

10-92 Maximizing the probability of rejecting H_0 is equivalent to minimizing

$$P\left(-z_{\alpha/2} < \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}} < z_{\alpha/2} \mid \mu_1 - \mu_2 = \delta\right) = P\left(-z_{\alpha/2} - \frac{\delta}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}} < Z < z_{\alpha/2} - \frac{\delta}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}\right)$$

where z is a standard normal random variable. This probability is minimized by maximizing $\frac{\delta}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$.

Therefore, we are to minimize $\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$ subject to $n_1 + n_2 = N$.

From the constraint, $n_2 = N - n_1$, and we are to minimize $f(n_1) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{N - n_1}$. Taking the derivative of $f(n_1)$ with respect to n_1 and setting it equal to zero results in the equation $\frac{-\sigma_1^2}{n_1^{\,2}} + \frac{\sigma_2^2}{(N-n_1)^2} = 0$.

Upon solving for n_1, we obtain $n_1 = \frac{\sigma_1 N}{\sigma_1 + \sigma_2}$ and $n_2 = \frac{\sigma_2 N}{\sigma_1 + \sigma_2}$

Also, it can be verified that the solution minimizes $f(n_1)$.

10-93 a) $\alpha = P(Z > z_\varepsilon \;\; or \;\; Z < -z_{\alpha-\varepsilon})$ where Z has a standard normal distribution.

Then, $\alpha = P(Z > z_\varepsilon) + P(Z < -z_{\alpha-\varepsilon}) = \varepsilon + \alpha - \varepsilon = \alpha$

b) $\beta = P(-z_{\alpha-\varepsilon} < Z_0 < z_\varepsilon | \mu_1 = \mu_0 + \delta)$

$$\beta = P(-z_{\alpha-\varepsilon} < \frac{\bar{x}-\mu_0}{\sqrt{\sigma^2/n}} < z_\varepsilon | \mu_1 = \mu_0 + \delta)$$

$$= P(-z_{\alpha-\varepsilon} - \frac{\delta}{\sqrt{\sigma^2/n}} < Z < z_\varepsilon - \frac{\delta}{\sqrt{\sigma^2/n}})$$

$$= \Phi(z_\varepsilon - \frac{\delta}{\sqrt{\sigma^2/n}}) - \Phi(-z_{\alpha-\varepsilon} - \frac{\delta}{\sqrt{\sigma^2/n}})$$

10-94. The requested result can be obtained from data in which the pairs are very different. Example:

pair	1	2	3	4	5
sample 1	100	10	50	20	70
sample 2	110	20	59	31	80

$\bar{x}_1 = 50 \quad \bar{x}_2 = 60$

$s_1 = 36.74 \qquad s_2 = 36.54 \qquad s_{pooled} = 36.64$

Two-sample t-test : $t_0 = -0.43$ P-value = 0.68

$\bar{x}_d = -10 \qquad s_d = 0.707$

Paired t-test : $t_0 = -31.62$ P-value ≈ 0

10-95 a.) $\theta = \dfrac{p_1}{p_2}$ and $\hat{\theta} = \dfrac{\hat{p}_1}{\hat{p}_2}$ and $\ln(\hat{\theta}) \sim N[\ln(\theta), \sqrt{(n_1 - x_1)/n_1 x_1 + (n_2 - x_2)/n_2 x_2}\,]$

The (1-α) confidence Interval for ln(θ) will use the relationship $Z = \dfrac{\ln(\hat{\theta}) - \ln(\theta)}{\left(\left(\dfrac{n_1 - x_1}{n_1 x_1}\right) + \left(\dfrac{n_2 - x_2}{n_2 x_2}\right)\right)^{1/4}}$

$$\ln(\hat{\theta}) - Z_{\alpha/2}\left(\left(\frac{n_1 - x_1}{n_1 x_1}\right) + \left(\frac{n_2 - x_2}{n_2 x_2}\right)\right)^{1/4} \le \ln(\theta) \le \ln(\hat{\theta}) + Z_{\alpha/2}\left(\left(\frac{n_1 - x_1}{n_1 x_1}\right) + \left(\frac{n_2 - x_2}{n_2 x_2}\right)\right)^{1/4}$$

b.) The (1-α) confidence Interval for θ use the CI developed in part (a.) where θ = e^(ln(θ))

$$\hat{\theta} - e^{Z_{\alpha/2}\left(\left(\frac{n_1 - x_1}{n_1 x_1}\right) + \left(\frac{n_2 - x_2}{n_2 x_2}\right)\right)^{1/4}} \le \theta \le \hat{\theta} + e^{Z_{\alpha/2}\left(\left(\frac{n_1 - x_1}{n_1 x_1}\right) + \left(\frac{n_2 - x_2}{n_2 x_2}\right)\right)^{1/4}}$$

c.)

$$\hat{\theta} - e^{Z_{\alpha/2}\left(\left(\frac{n_1 - x_1}{n_1 x_1}\right) + \left(\frac{n_2 - x_2}{n_2 x_2}\right)\right)^{.25}} \le \theta \le \hat{\theta} + e^{Z_{\alpha/2}\left(\left(\frac{n_1 - x_1}{n_1 x_1}\right) + \left(\frac{n_2 - x_2}{n_2 x_2}\right)\right)^{.25}}$$

$$1.42 - e^{1.96\left(\left(\frac{100-27}{2700}\right) + \left(\frac{100-19}{1900}\right)\right)^{1/4}} \le \theta \le 1.42 + e^{1.96\left(\left(\frac{100-27}{2700}\right) + \left(\frac{100-19}{1900}\right)\right)^{1/4}}$$

$$-1.317 \le \theta \le 4.157$$

Since the confidence interval contains the value 1, we conclude that there is no difference in the proportions at the 95% level of significance

10-96 $H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$

$$\beta = P\left(f^2_{1-\alpha/2, n_1-1, n_2-1} < \frac{S_1^2}{S_2^2} < f^2_{\alpha/2, n_1-1, n_2-1} \mid \frac{\sigma_1^2}{\sigma_2^2} = \delta \neq 1 \right)$$

$$= P\left(\frac{\sigma_2^2}{\sigma_1^2} f_{1-\alpha/2, n_1-1, n_2-1} < \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} < \frac{\sigma_2^2}{\sigma_1^2} f_{\alpha/2, n_1-1, n_2-1} \mid \frac{\sigma_1^2}{\sigma_2^2} = \delta \right)$$

where $\frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2}$ has an F distribution with $n_1 - 1$ and $n_2 - 1$ degrees of freedom.