CHAPTER 9

Section 9-1

9-1 a) $H_0 : \mu = 25, H_1 : \mu \neq 25$ Yes, because the hypothesis is stated in terms of the parameter of interest, inequality is in the alternative hypothesis, and the value in the null and alternative hypotheses matches.

b) $H_0 : \sigma > 10, H_1 : \sigma = 10$ No, because the inequality is in the null hypothesis.

c) $H_0 : \bar{x} = 50, H_1 : \bar{x} \neq 50$ No, because the hypothesis is stated in terms of the statistic rather than the parameter.

d) $H_0 : p = 0.1, H_1 : p = 0.3$ No, the values in the null and alternative hypotheses do not match and both of the hypotheses are equality statements.

e) $H_0 : s = 30, H_1 : s > 30$ No, because the hypothesis is stated in terms of the statistic rather than the parameter.

9-2 a) α = P(reject H_0 when H_0 is true)

= P($\bar{X} \leq 11.5$ when $\mu = 12$) = $P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \leq \frac{11.5-12}{0.5/\sqrt{4}}\right)$ = P(Z ≤ −2)

= 0.02275.

The probability of rejecting the null hypothesis when it is true is 0.02275.

b) β = P(accept H_0 when $\mu = 11.25$) = $P(\bar{X} > 11.5 \text{ when } \mu = 11.25)$ = $P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{11.5-11.25}{0.5/\sqrt{4}}\right)$

P(Z > 1.0) = 1 − P(Z ≤ 1.0) = 1 − 0.84134 = 0.15866

The probability of accepting the null hypothesis when it is false is 0.15866.

9-3 a) $\alpha = P(\bar{X} \leq 11.5 \mid \mu = 12) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \leq \frac{11.5-12}{0.5/\sqrt{16}}\right)$ = P(Z ≤ −4) = 0.

The probability of rejecting the null, when the null is true, is approximately 0 with a sample size of 16.

b) $\beta = P(\bar{X} > 11.5 \mid \mu = 11.25) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{11.5-11.25}{0.5/\sqrt{16}}\right)$ = P(Z > 2) = 1 − P(Z ≤ 2)

= 1− 0.97725 = 0.02275.

The probability of accepting the null hypothesis when it is false is 0.02275.

9-4 Find the boundary of the critical region if $\alpha = 0.01$:

$$0.01 = P\left(Z \leq \frac{c-12}{0.5/\sqrt{4}}\right)$$

What Z value will give a probability of 0.01? Using Table 2 in the appendix, Z value is −2.33.

Thus, $\frac{c-12}{0.5/\sqrt{4}} = -2.33$, c = 11.4175

9-5. $P\left(Z \le \frac{c-12}{0.5/\sqrt{4}}\right) = 0.05$

What Z value will give a probability of 0.05? Using Table 2 in the appendix, Z value is −1.65.

Thus, $\frac{c-12}{0.5/\sqrt{4}} = -1.65$, c = 11.5875

9-6 a) $\alpha = P(\overline{X} \le 98.5) + P(\overline{X} > 101.5)$

$$= P\left(\frac{\overline{X}-100}{2/\sqrt{9}} \le \frac{98.5-100}{2/\sqrt{9}}\right) + P\left(\frac{\overline{X}-100}{2/\sqrt{9}} > \frac{101.5-100}{2/\sqrt{9}}\right)$$

$= P(Z \le -2.25) + P(Z > 2.25)$
$= (P(Z \le -2.25)) + (1 - P(Z \le 2.25))$
$= 0.01222 + 1 - 0.98778$
$= 0.01222 + 0.01222 = 0.02444$

b) $\beta = P(98.5 \le \overline{X} \le 101.5$ when $\mu = 103)$

$$= P\left(\frac{98.5-103}{2/\sqrt{9}} \le \frac{\overline{X}-103}{2/\sqrt{9}} \le \frac{101.5-103}{2/\sqrt{9}}\right)$$

$= P(-6.75 \le Z \le -2.25)$
$= P(Z \le -2.25) - P(Z \le -6.75)$
$= 0.01222 - 0 = 0.01222$

c) $\beta = P(98.5 \le \overline{X} \le 101.5$ when $\mu = 105)$

$$= P\left(\frac{98.5-105}{2/\sqrt{9}} \le \frac{\overline{X}-105}{2/\sqrt{9}} \le \frac{101.5-105}{2/\sqrt{9}}\right)$$

$= P(-9.75 \le Z \le -5.25)$
$= P(Z \le -5.25) - P(Z \le -9.75)$
$= 0 - 0$
$= 0.$

The probability of accepting the null hypothesis when it is actually false is smaller in part c since the true mean, $\mu = 105$, is further from the acceptance region. A larger difference exists.

9-7 Use $n = 5$, everything else held constant (from the values in exercise 9-6):

a) $P(\overline{X} \le 98.5) + P(\overline{X} > 101.5)$

$$= P\left(\frac{\overline{X}-100}{2/\sqrt{5}} \le \frac{98.5-100}{2/\sqrt{5}}\right) + P\left(\frac{\overline{X}-100}{2/\sqrt{5}} > \frac{101.5-100}{2/\sqrt{5}}\right)$$

$= P(Z \le -1.68) + P(Z > 1.68)$
$= P(Z \le -1.68) + (1 - P(Z \le 1.68))$
$= 0.04648 + (1 - 0.95352)$
$= 0.09296$

b) $\beta = P(98.5 \le \overline{X} \le 101.5$ when $\mu = 103)$

$$= P\left(\frac{98.5-103}{2/\sqrt{5}} \le \frac{\overline{X}-103}{2/\sqrt{5}} \le \frac{101.5-103}{2/\sqrt{5}}\right)$$

$= P(-5.03 \le Z \le -1.68)$
$= P(Z \le -1.68) - P(Z \le -5.03)$
$= 0.04648 - 0$
$= 0.04648$

c) β = P(98.5 ≤ $\bar{x}$ ≤ 101.5 when μ = 105)

$$= P\left(\frac{98.5-105}{2/\sqrt{5}} \le \frac{\bar{X}-105}{2/\sqrt{5}} \le \frac{101.5-105}{2/\sqrt{5}}\right)$$

= P(−7.27≤ Z ≤ −3.91)
= P(Z ≤ −3.91) − P(Z ≤ −7.27)
= 0.00005 − 0
= 0.00005
It is smaller, because it is not likely to accept the product when the true mean is as high as 105.

9-8 a) α = P($\bar{X}$ > 185 when μ = 175)

$$= P\left(\frac{\bar{X}-175}{20/\sqrt{10}} > \frac{185-175}{20/\sqrt{10}}\right)$$

= P(Z > 1.58)
= 1 − P(Z ≤ 1.58)
= 1 − 0.94295
= 0.05705

b) β = P($\bar{X}$ ≤ 185 when μ = 195)

$$= P\left(\frac{\bar{X}-195}{20/\sqrt{10}} \le \frac{185-195}{20/\sqrt{10}}\right)$$

= P(Z ≤ −1.58)
= 0.05705.

9-9 a) $z = \dfrac{190-175}{20/\sqrt{10}} = 2.37$, Note that z is large, therefore **reject** the null hypothesis and conclude that the mean foam height is greater than 175 mm.

b) P($\bar{X}$ > 190 when μ = 175)

$$= P\left(\frac{\bar{X}-175}{20/\sqrt{10}} > \frac{190-175}{20/\sqrt{10}}\right)$$

= P(Z > 2.37) = 1 − P(Z ≤ 2.37)
= 1 − 0.99111
= 0.00889.

The probability that a value of at least 190 mm would be observed (if the true mean height is 175 mm) is only 0.00889. Thus, the sample value of $\bar{x}$ = 190 mm would be an unusual result.

9-10 Using n = 16:

a) α = P($\bar{X}$ > 185 when μ = 175)

$$= P\left(\frac{\bar{X}-175}{20/\sqrt{16}} > \frac{185-175}{20/\sqrt{16}}\right)$$

= P(Z > 2)
= 1 − P(Z ≤ 2)
= 1 − 0.97725
= 0.02275

b) β = P($\overline{X}$ ≤ 185 when μ = 195)

$$= P\left(\frac{\overline{X}-195}{20/\sqrt{16}} \le \frac{185-195}{20/\sqrt{16}}\right)$$

= P(Z ≤ −2)
= 0.02275.

9-11 a) $P(\overline{X} > c \mid \mu = 175) = 0.0571$

$$P\left(Z > \frac{c-175}{20/\sqrt{16}}\right) = P(Z \ge 1.58)$$

Thus, $1.58 = \dfrac{c-175}{20/\sqrt{16}}$, and $c = 182.9$

b) If the true mean foam height is 195 mm, then

β = P($\overline{X}$ ≤ 182.9 when μ = 195)

$$= P\left(Z \le \frac{182.9-195}{20/\sqrt{16}}\right)$$

= P(Z ≤ −2.42)
= 0.00776

c) For the same level of α, with the increased sample size, β is reduced.

9-12 a) α = P($\overline{X}$ ≤ 4.85 when μ = 5) + P($\overline{X}$ > 5.15 when μ = 5)

$$= P\left(\frac{\overline{X}-5}{0.25/\sqrt{8}} \le \frac{4.85-5}{0.25/\sqrt{8}}\right) + P\left(\frac{\overline{X}-5}{0.25/\sqrt{8}} > \frac{5.15-5}{0.25/\sqrt{8}}\right)$$

= P(Z ≤ −1.7) + P(Z > 1.7)
= P(Z ≤ −1.7) + (1 − P(Z ≤ 1.7)
= 0.04457 + (1 − 0.95543)
= 0.08914.

b) Power = 1 − β

β = P(4.85 ≤ $\overline{X}$ ≤ 5.15 when μ = 5.1)

$$= P\left(\frac{4.85-5.1}{0.25/\sqrt{8}} \le \frac{\overline{X}-5.1}{0.25/\sqrt{8}} \le \frac{5.15-5.1}{0.25/\sqrt{8}}\right)$$

= P(−2.83 ≤ Z ≤ 0.566)
= P(Z ≤ 0.566) − P(Z ≤ −2.83)
= 0.71566 − 0.00233
= 0.71333
1 − β = 0.2867.

9-13 Using n = 16:

a) $\alpha = P(\overline{X} \le 4.85 \mid \mu = 5) + P(\overline{X} > 5.15 \mid \mu = 5)$

$$= P\left(\frac{\overline{X}-5}{0.25/\sqrt{16}} \le \frac{4.85-5}{0.25/\sqrt{16}}\right) + P\left(\frac{\overline{X}-5}{0.25/\sqrt{16}} > \frac{5.15-5}{0.25/\sqrt{16}}\right)$$

$= P(Z \le -2.4) + P(Z > 2.4)$
$= P(Z \le -2.4) + (1 - P(Z \le 2.4))$
$= 2(1 - P(Z \le 2.4))$
$= 2(1 - 0.99180)$
$= 2(0.0082)$
$= 0.0164.$

b) $\beta = P(4.85 \le \overline{X} \le 5.15 \mid \mu = 5.1)$

$$= P\left(\frac{4.85-5.1}{0.25/\sqrt{16}} \le \frac{\overline{X}-5.1}{0.25/\sqrt{16}} \le \frac{5.15-5.1}{0.25/\sqrt{16}}\right)$$

$= P(-4 \le Z \le 0.8) = P(Z \le 0.8) - P(Z \le -4)$
$= 0.78814 - 0$
$= 0.78814$
$1 - \beta = 0.21186$

9-14 Find the boundary of the critical region if $\alpha = 0.05$:

$$0.025 = P\left(Z \le \frac{c-5}{0.25/\sqrt{8}}\right)$$

What Z value will give a probability of 0.025? Using Table 2 in the appendix, Z value is −1.96.

Thus, $\frac{c-5}{0.25/\sqrt{8}} = -1.96$, c = 4.83 and

$$\frac{c-5}{0.25/\sqrt{8}} = 1.96, \quad c=5.17$$

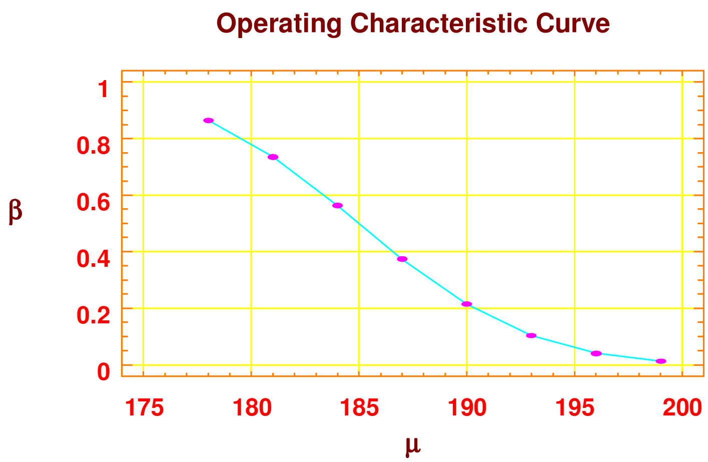
The acceptance region should be $(4.83 \le \overline{X} \le 5.17)$.

9-15 Operating characteristic curve:

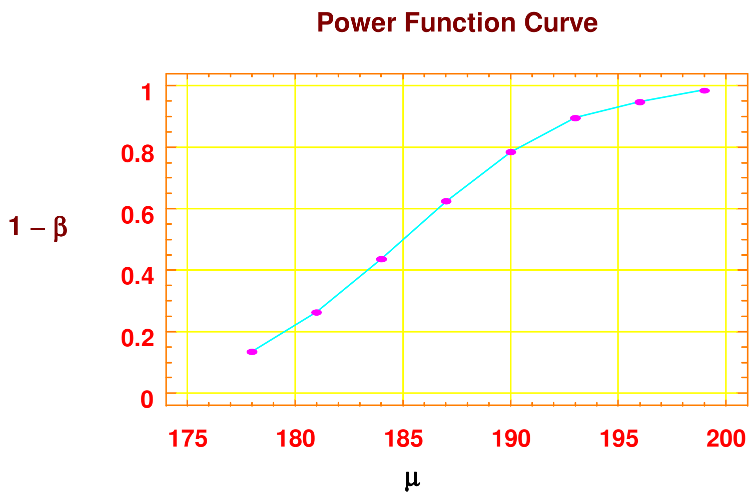
$\overline{x} = 185$

$$\beta = P\left(Z \le \frac{\overline{x}-\mu}{20/\sqrt{10}}\right) = P\left(Z \le \frac{185-\mu}{20/\sqrt{10}}\right)$$

μ	$P\left(Z \le \frac{185-\mu}{20/\sqrt{10}}\right) =$	β	$1-\beta$
178	$P(Z \le 1.11) =$	0.8665	0.1335
181	$P(Z \le 0.63) =$	0.7357	0.2643
184	$P(Z \le 0.16) =$	0.5636	0.4364
187	$P(Z \le -0.32) =$	0.3745	0.6255
190	$P(Z \le -0.79) =$	0.2148	0.7852
193	$P(Z \le -1.26) =$	0.1038	0.8962
196	$P(Z \le -1.74) =$	0.0409	0.9591
199	$P(Z \le -2.21) =$	0.0136	0.9864



9-16



9-17. The problem statement implies H_0: $p = 0.6$, H_1: $p > 0.6$ and defines an acceptance region as

$$\hat{p} \leq \frac{400}{500} = 0.80 \text{ and rejection region as } \hat{p} > 0.80$$

a) $\alpha = P(\hat{p} > 0.80 \mid p = 0.60) = P\left(Z > \dfrac{0.80 - 0.60}{\sqrt{\dfrac{0.6(0.4)}{500}}} \right)$

$= P(Z > 9.13) = 1 - P(Z \leq 9.13) \approx 0$

b) $\beta = P(\hat{p} \leq 0.8 \text{ when } p = 0.75) = P(Z \leq 2.58) = 0.99506$.

9-18 $X \sim \text{bin}(10, 0.3)$ Implicitly, H_0: $p = 0.3$ and H_1: $p < 0.3$
$n = 10$
Accept region: $\hat{p} > 0.1$
Reject region: $\hat{p} \le 0.1$
Use the normal approximation for parts a), b) and c):

a) When p =0.3 α = $P(\hat{p} < 0.1) = P\left(Z \le \dfrac{0.1-0.3}{\sqrt{\dfrac{0.3(0.7)}{10}}}\right)$

$$= P(Z \le -1.38)$$
$$= 0.08379$$

b) When p = 0.2 β = $P(\hat{p} > 0.1) = P\left(Z > \dfrac{0.1-0.2}{\sqrt{\dfrac{0.2(0.8)}{10}}}\right)$

$$= P(Z > -0.79)$$
$$= 1 - P(Z < -0.79)$$
$$= 0.78524$$

c) Power = 1 − β = 1 − 078524 = 0.21476

9-19 X ~ bin(15, 0.4) H_0: $p = 0.4$ and H_1: $p \ne 0.4$

p_1= 4/15 = 0.267 p_2 = 8/15 = 0.533

Accept Region: $0.267 \le \hat{p} \le 0.533$
Reject Region: $\hat{p} < 0.267$ or $\hat{p} > 0.533$

Use the normal approximation for parts a) and b)

a) When p = 0.4, $\alpha = P(\hat{p} < 0.267) + P(\hat{p} > 0.533)$

$$= P\left(Z < \dfrac{0.267 - 0.4}{\sqrt{\dfrac{0.4(0.6)}{15}}}\right) + P\left(Z > \dfrac{0.533 - 0.4}{\sqrt{\dfrac{0.4(0.6)}{15}}}\right)$$
$$= P(Z < -1.05) + P(Z > 1.05)$$
$$= P(Z < -1.05) + (1 - P(Z < 1.05))$$
$$= 0.14686 + 0.14686$$
$$= 0.29372$$

b) When p = 0.2,

$$\beta = P(0.267 \le \hat{p} \le 0.533) = P\left(\frac{0.267-0.2}{\sqrt{\frac{0.2(0.8)}{15}}} \le Z \le \frac{0.533-0.2}{\sqrt{\frac{0.2(0.8)}{15}}}\right)$$
$$= P(0.65 \le Z \le 3.22)$$
$$= P(Z \le 3.22) - P(Z \le 0.65)$$
$$= 0.99936 - 0.74215$$
$$= 0.25721$$

<u>Section 9-2</u>

9-20 a.) 1) The parameter of interest is the true mean water temperature, μ.

2) $H_0 : \mu = 100$

3) $H_1 : \mu > 100$

4) $\alpha = 0.05$

5) $z_0 = \dfrac{\overline{x} - \mu}{\sigma / \sqrt{n}}$

6) Reject H_0 if $z_0 > z_\alpha$ where $z_{0.05} = 1.65$

7) $\overline{x} = 98$, $\sigma = 2$

$$z_0 = \frac{98-100}{2/\sqrt{9}} = -3.0$$

8) Since -3.0 < 1.65 do not reject H_0 and conclude the water temperature is not significantly different greater than 100 at $\alpha = 0.05$.

b) P-value = $1 - \Phi(-3.0) = 1 - 0.00135 = 0.99865$

c) $\beta = \Phi\left(z_{0.05} + \dfrac{100-104}{2/\sqrt{9}}\right)$

$= \Phi(1.65 + -6)$

$= \Phi(-4.35)$

$\cong 0$

9-21. a) 1) The parameter of interest is the true mean yield, μ.

2) $H_0 : \mu = 90$

3) $H_1 : \mu \ne 90$

4) $\alpha = 0.05$

5) $z_0 = \dfrac{\overline{x} - \mu}{\sigma / \sqrt{n}}$

6) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $-z_{0.025} = -1.96$ or $z_0 > z_{\alpha/2}$ where $z_{0.025} = 1.96$

7) $\overline{x} = 90.48$, $\sigma = 3$

$$z_0 = \frac{90.48-90}{3/\sqrt{5}} = 0.36$$

8) Since $-1.96 < 0.36 < 1.96$ do not reject H_0 and conclude the yield is not significantly different from 90% at $\alpha = 0.05$.

b) P-value = $2[1 - \Phi(0.36)] = 2[1 - 0.64058] = 0.71884$

c) $n = \dfrac{(z_{\alpha/2} + z_\beta)^2 \sigma^2}{\delta^2} = \dfrac{(z_{0.025} + z_{0.05})^2 3^2}{(85-90)^2} = \dfrac{(1.96 + 1.65)^2 9}{(-5)^2} = 4.69$

$n \cong 5$.

d) $\beta = \Phi\left(z_{0.025} + \frac{90-92}{3/\sqrt{5}}\right) - \Phi\left(-z_{0.025} + \frac{90-92}{3/\sqrt{5}}\right)$

$= \Phi(1.96 + -1.491) - \Phi(-1.96 + \ -1.491)$
$= \Phi(0.47) - \Phi(-3.45)$
$= \Phi(0.47) - (1 - \Phi(3.45))$
$= 0.68082 - (\ 1 - 0.99972)$
$= 0.68054.$

e) For $\alpha = 0.05$, $z_{\alpha/2} = z_{0.025} = 1.96$

$$\bar{x} - z_{0.025}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu \le \bar{x} + z_{0.025}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$90.48 - 1.96\left(\frac{3}{\sqrt{5}}\right) \le \mu \le 90.48 + 1.96\left(\frac{3}{\sqrt{5}}\right)$$

$87.85 \le \mu \le 93.11$

With 95% confidence, we believe the true mean yield of the chemical process is between 87.85% and 93.11%. Since 90% is contained in the confidence interval, our decision in (a) agrees with the confidence interval.

9-22 a) 1) The parameter of interest is the true mean crankshaft wear, μ.
2) $H_0 : \mu = 3$
3) $H_1 : \mu \ne 3$
4) $\alpha = 0.05$
5) $z_0 = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$
6)) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $-z_{0.025} = -1.96$ or $z_0 > z_{\alpha/2}$ where $z_{0.025} = 1.96$
7) $\bar{x} = 2.78$, $\sigma = 0.9$

$$z_0 = \frac{2.78 - 3}{0.9/\sqrt{15}} = -0.95$$

8) Since $-0.95 > -1.96$, do not reject the null hypothesis and conclude there is not sufficient evidence to support the claim the mean crankshaft wear is not equal to 3 at $\alpha = 0.05$.

b) $\beta = \Phi\left(z_{0.025} + \frac{3 - 3.25}{0.9/\sqrt{15}}\right) - \Phi\left(-z_{0.025} + \frac{3 - 3.25}{0.9/\sqrt{15}}\right)$

$= \Phi(1.96 + -1.08) - \Phi(-1.96 + \ -1.08)$
$= \Phi(0.88) - \Phi(-3.04)$
$= 0.81057 - (0.00118)$
$= 0.80939$

c.) $n = \frac{(z_{\alpha/2} + z_\beta)^2 \sigma^2}{\delta^2} = \frac{(z_{0.025} + z_{0.10})^2 \sigma^2}{(3.75 - 3)^2} = \frac{(1.96 + 1.29)^2 (0.9)^2}{(0.75)^2} = 15.21,$

$n \cong 16$

9-23. a) 1) The parameter of interest is the true mean melting point, μ.

2) $H_0 : \mu = 155$

3) $H_1 : \mu \neq 155$

4) $\alpha = 0.01$

5) $z_0 = \dfrac{\overline{x} - \mu}{\sigma / \sqrt{n}}$

6) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $-z_{0.005} = -2.58$ or $z_0 > z_{\alpha/2}$ where $z_{0.005} = 2.58$

7) $\overline{x} = 154.2$, $\sigma = 1.5$

$$z_0 = \frac{154.2 - 155}{1.5/\sqrt{10}} = -1.69$$

8) Since $-1.69 > -2.58$, do not reject the null hypothesis and conclude there is not sufficient evidence to support the claim the mean melting point is not equal to 155 °F at $\alpha = 0.01$.

b) P-value = 2*P(Z <- 1.69) =2* 0.045514 = 0.091028

c)

$$\beta = \Phi\left(z_{0.005} - \frac{\delta\sqrt{n}}{\sigma} \right) - \Phi\left(-z_{0.005} - \frac{\delta\sqrt{n}}{\sigma} \right)$$

$$= \Phi\left(2.58 - \frac{(155-150)\sqrt{10}}{1.5} \right) - \Phi\left(-2.58 - \frac{(155-150)\sqrt{10}}{1.5} \right)$$

$= \Phi(-7.96) - \Phi(-13.12) = 0 - 0 = 0$

d)

$$n = \frac{\left(z_{\alpha/2} + z_\beta\right)^2 \sigma^2}{\delta^2} = \frac{\left(z_{0.005} + z_{0.10}\right)^2 \sigma^2}{(150-155)^2} = \frac{(2.58+1.29)^2 (1.5)^2}{(5)^2} = 1.35,$$

$n \cong 2$.

9-24 a.) 1) The parameter of interest is the true mean battery life in hours, μ.

2) $H_0 : \mu = 40$

3) $H_1 : \mu > 40$

4) $\alpha = 0.05$

5) $z_0 = \dfrac{\overline{x} - \mu}{\sigma / \sqrt{n}}$

6) Reject H_0 if $z_0 > z_\alpha$ where $z_{0.05} = 1.65$

7) $\overline{x} = 40.5$, $\sigma = 1.25$

$$z_0 = \frac{40.5 - 40}{1.25/\sqrt{10}} = 1.26$$

8) Since $1.26 < 1.65$ do not reject H_0 and conclude the battery life is not significantly different greater than 40 at $\alpha = 0.05$.

b) P-value = $1 - \Phi(1.26) = 1 - 0.8962 = 0.1038$

c) $\beta = \Phi\left(z_{0.05} + \dfrac{40-42}{1.25/\sqrt{10}} \right)$

$= \Phi(1.65 + -5.06)$

$= \Phi(-3.41)$

$\cong 0.000325$

d.) $n = \dfrac{\left(z_\alpha + z_\beta\right)^2 \sigma^2}{\delta^2} = \dfrac{\left(z_{0.05} + z_{0.10}\right)^2 \sigma^2}{(40-44)^2} = \dfrac{(1.65+1.29)^2 (1.25)^2}{(4)^2} = 0.844,$

$n \cong 1$

e.)95% Confidence Interval

$$\bar{x} + z_{0.05}\sigma/\sqrt{n} \le \mu$$
$$40.5 + 1.65(1.25)/\sqrt{10} \le \mu$$
$$39.85 \le \mu$$

The lower bound of the 90 % confidence interval must be greater than 40 to verify that the true mean exceeds 40 hours.

9-25. a) 1) The parameter of interest is the true mean tensile strength, μ.

2) $H_0 : \mu = 3500$

3) $H_1 : \mu \neq 3500$

4) $\alpha = 0.01$

5) $z_0 = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

6) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $-z_{0.005} = -2.58$ or $z_0 > z_{\alpha/2}$ where $z_{0.005} = 2.58$

7) $\bar{x} = 3250$, $\sigma = 60$

$$z_0 = \frac{3250 - 3500}{60/\sqrt{12}} = -14.43$$

8) Since $-14.43 < -2.58$, reject the null hypothesis and conclude the true mean tensile strength is significantly different from 3500 at $\alpha = 0.01$.

b) Smallest level of significance = P-value = $2[1 - \Phi(14.43)] = 2[1 - 1] = 0$

The smallest level of significance at which we are willing to reject the null hypothesis is 0.

c) $z_{\alpha/2} = z_{0.005} = 2.58$

$$\bar{x} - z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu \le \bar{x} + z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$3250 - 2.58\left(\frac{31.62}{\sqrt{12}}\right) \le \mu \le 3250 + 2.58\left(\frac{31.62}{\sqrt{12}}\right)$$

$3205.31 \le \mu \le 3294.69$

With 95% confidence, we believe the true mean tensile strength is between 3205.31 psi and 3294.69 psi. We can test the hypotheses that the true mean tensile strength is not equal to 3500 by noting that the value is not within the confidence interval.

9-26
$$n = \frac{(z_{\alpha/2} + z_\beta)^2\sigma^2}{\delta^2} = \frac{(z_{0.05} + z_{0.20})^2\sigma^2}{(3250 - 3500)^2} = \frac{(1.65 + .84)^2(60)^2}{(250)^2} = 0.357,$$
$$n \cong 1$$

9-27 a) 1) The parameter of interest is the true mean speed, μ.

2) $H_0 : \mu = 100$

3) $H_1 : \mu < 100$

4) $\alpha = 0.05$

5) $z_0 = \dfrac{\overline{x} - \mu}{\sigma / \sqrt{n}}$

6) Reject H_0 if $z_0 < -z_\alpha$ where $-z_{0.05} = -1.65$

7) $\overline{x} = 102.2$, $\sigma = 4$

$$z_0 = \frac{102.2 - 100}{4/\sqrt{8}} = 1.56$$

8) Since 1.56> −1.65, do not reject the null hypothesis and conclude the there is insufficient evidence to conclude that the true speed strength is less than 100 at $\alpha = 0.05$.

b) $\beta = \Phi\left(-z_{0.05} - \dfrac{(95-100)\sqrt{8}}{4} \right)$ = Φ(-1.65 - −3.54) = Φ(1.89) = 0.97062

Power = 1-β = 1-0.97062 = 0.02938

c) n = $\dfrac{(z_\alpha + z_\beta)^2 \sigma^2}{\delta^2} = \dfrac{(z_{0.05} + z_{0.15})^2 \sigma^2}{(95-100)^2} = \dfrac{(1.65 + 1.03)^2 (4)^2}{(5)^2} = 4.597,$

$n \cong 5$

d) $\overline{x} - z_{0.05}\left(\dfrac{\sigma}{\sqrt{n}} \right) \le \mu$

$$102.2 - 1.65\left(\frac{4}{\sqrt{8}} \right) \le \mu$$

$$99.866 \le \mu$$

Since the lower limit of the CI is just slightly below 100, we are relatively confident that the mean speed is not less than 100 m/s. Also the sample mean is greater than 100.

9-28 a) 1) The parameter of interest is the true mean hole diameter, μ.

2) $H_0 : \mu = 1.50$

3) $H_1 : \mu \neq 1.50$

4) $\alpha = 0.01$

5) $z_0 = \dfrac{\overline{x} - \mu}{\sigma / \sqrt{n}}$

6) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $-z_{0.005} = -2.58$ or $z_0 > z_{\alpha/2}$ where $z_{0.005} = 2.58$

7) $\overline{x} = 1.4975$, $\sigma = 0.01$

$$z_0 = \frac{1.4975 - 1.50}{0.01/\sqrt{25}} = -1.25$$

8) Since −2.58 < -1.25 < 2.58, do not reject the null hypothesis and conclude the true mean hole diameter is not significantly different from 1.5 in. at $\alpha = 0.01$.

b)

$$\beta = \Phi\left(z_{0.005} - \frac{\delta\sqrt{n}}{\sigma}\right) - \Phi\left(-z_{0.005} - \frac{\delta\sqrt{n}}{\sigma}\right)$$

$$= \Phi\left(2.58 - \frac{(1.495-1.5)\sqrt{25}}{0.01}\right) - \Phi\left(-2.58 - \frac{(1.495-1.5)\sqrt{25}}{0.01}\right)$$

$= \Phi(5.08)- \Phi(-0.08) = 1 - .46812 = 0.53188$

power=1-β=0.46812.

c) Set $\beta = 1 - 0.90 = 0.10$

$$n = \frac{(z_{\alpha/2} + z_{\beta})^2\sigma^2}{\delta^2} = \frac{(z_{0.005} + z_{0.10})^2\sigma^2}{(1.495-1.50)^2} \cong \frac{(2.58+1.29)^2(0.01)^2}{(-0.005)^2} = 59.908,$$

$n \cong 60$.

d) For $\alpha = 0.01$, $z_{\alpha/2} = z_{0.005} = 2.58$

$$\bar{x} - z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu \le \bar{x} + z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$1.4975 - 2.58\left(\frac{0.01}{\sqrt{25}}\right) \le \mu \le 1.4975 + 2.58\left(\frac{0.01}{\sqrt{25}}\right)$$

$1.4923 \le \mu \le 1.5027$

The confidence interval constructed contains the value 1.5, thus the true mean hole diameter could possibly be 1.5 in. using a 99% level of confidence. Since a two-sided 99% confidence interval is equivalent to a two-sided hypothesis test at $\alpha = 0.01$, the conclusions necessarily must be consistent.

9-29 a) 1) The parameter of interest is the true average battery life, μ.

2) $H_0 : \mu = 4$

3) $H_1 : \mu > 4$

4) $\alpha = 0.05$

5) $z_0 = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

6) Reject H_0 if $z_0 > z_\alpha$ where $z_{0.05} = 1.65$

7) $\bar{x} = 4.05$, $\sigma = 0.2$

$$z_0 = \frac{4.05-4}{0.2/\sqrt{50}} = 1.77$$

8) Since 1.77>1.65, reject the null hypothesis and conclude that there is sufficient evidence to conclude that the true average battery life exceeds 4 hours at $\alpha = 0.05$.

b) $\beta = \Phi\left(z_{0.05} - \dfrac{(4.5-4)\sqrt{50}}{0.2}\right) = \Phi(1.65 - 17.68) = \Phi(-16.03) = 0$

Power = 1-β = 1-0 = 1

c) $n = \dfrac{(z_\alpha + z_\beta)^2\sigma^2}{\delta^2} = \dfrac{(z_{0.05} + z_{0.1})^2\sigma^2}{(4.5-4)^2} = \dfrac{(1.65+1.29)^2(0.2)^2}{(0.5)^2} = 1.38,$

$n \cong 2$

d) $\bar{x} - z_{0.05}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu$

$$4.05 - 1.65\left(\frac{0.2}{\sqrt{50}}\right) \le \mu$$

$$4.003 \le \mu$$

Since the lower limit of the CI is just slightly above 4, we conclude that average life is greater than 4 hours at α=0.05.

Section 9-3

9-30 a. 1) The parameter of interest is the true mean interior temperature life, μ.

2) $H_0 : \mu = 22.5$

3) $H_1 : \mu \ne 22.5$

4) $\alpha = 0.05$

5) $t_0 = \frac{\bar{x} - \mu}{s / \sqrt{n}}$

6) Reject H_0 if $|t_0| > t_{\alpha/2,n-1}$ where $t_{\alpha/2,n-1} = 2.776$

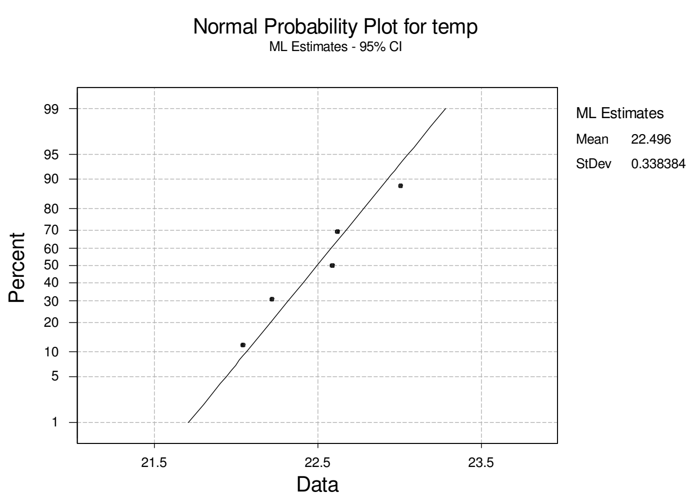
7) $\bar{x} = 22.496$, $s = 0.378$ n=5

$$t_0 = \frac{22.496 - 22.5}{0.378 / \sqrt{5}} = -0.00237$$

8) Since –0.00237 >- 2.776, we cannot reject the null hypothesis. There is not sufficient evidence to conclude that the true mean interior temperature is not equal to 22.5 °C at $\alpha = 0.05$.

2*0.4 <*P-value* < 2* 0.5 ; 0..8 < *P*-value <1.0

b.) The points on the normal probability plot fall along the line. Therefore, there is no evidence to conclude that the interior temperature data is not normally distributed.



c.) $d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|22.75 - 22.5|}{0.378} = 0.66$

Using the OC curve, Chart VI e) for $\alpha = 0.05$, $d = 0.66$, and n = 5, we get $\beta \cong 0.8$ and power of 1–0.8 = 0.2.

d) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|22.75 - 22.5|}{0.378} = 0.66$

Using the OC curve, Chart VI e) for $\alpha = 0.05$, $d = 0.66$, and $\beta \cong 0.1$ (Power=0.9),
$n = 40$.

e) 95% two sided confidence interval

$$\bar{x} - t_{0.025,4}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,4}\left(\frac{s}{\sqrt{n}}\right)$$

$$22.496 - 2.776\left(\frac{0.378}{\sqrt{5}}\right) \le \mu \le 22.496 + 2.776\left(\frac{0.378}{\sqrt{5}}\right)$$

$$22.027 \le \mu \le 22.965$$

We cannot conclude that the mean interior temperature is not equal to 22.5 since the value is included inside the confidence interval.

9-31 a. 1) The parameter of interest is the true mean female body temperature, μ.

2) $H_0 : \mu = 98.6$

3) $H_1 : \mu \neq 98.6$

4) $\alpha = 0.05$

5) $t_0 = \dfrac{\bar{x} - \mu}{s / \sqrt{n}}$

6) Reject H_0 if $|t_0| > t_{\alpha/2,n-1}$ where $t_{\alpha/2,n-1} = 2.064$

7) $\bar{x} = 98.264$, s = 0.4821 n=25

$$t_0 = \frac{98.264 - 98.6}{0.4821/\sqrt{25}} = -3.48$$

8) Since 3.48 > 2.064, reject the null hypothesis and conclude that the there is sufficient evidence to conclude that the true mean female body temperature is not equal to 98.6 °F at $\alpha = 0.05$.
P-value = 2* 0.001 = 0.002

b) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|98 - 98.6|}{0.4821} = 1.24$

Using the OC curve, Chart VI e) for $\alpha = 0.05$, d = 1.24, and n = 25, we get $\beta \cong 0$ and power of $1-0 \cong 1$.

c) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|98.2 - 98.6|}{0.4821} = 0.83$

Using the OC curve, Chart VI e) for $\alpha = 0.05$, d = 0.83, and $\beta \cong 0.1$ (Power=0.9),
$n = 20$.

d) 95% two sided confidence interval

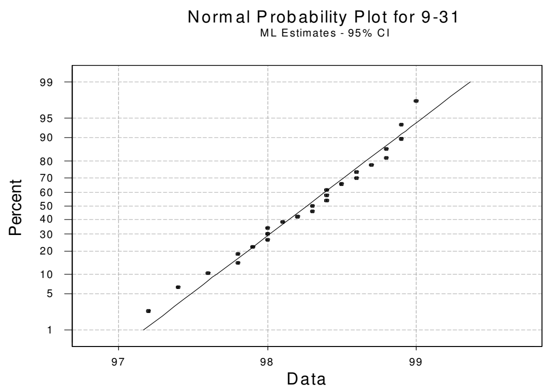
$$\bar{x} - t_{0.025,24}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,24}\left(\frac{s}{\sqrt{n}}\right)$$

$$98.264 - 2.064\left(\frac{0.4821}{\sqrt{25}}\right) \le \mu \le 98.264 + 2.064\left(\frac{0.4821}{\sqrt{25}}\right)$$

$$98.065 \le \mu \le 98.463$$

We can conclude that the mean female body temperature is not equal to 98.6 since the value is not included inside the confidence interval.

e)



Data appear to be normally distributed.

9-32 a.) 1) The parameter of interest is the true mean rainfall, μ.

2) $H_0 : \mu = 25$

3) $H_1 : \mu > 25$

4) $\alpha = 0.01$

5) $t_0 = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

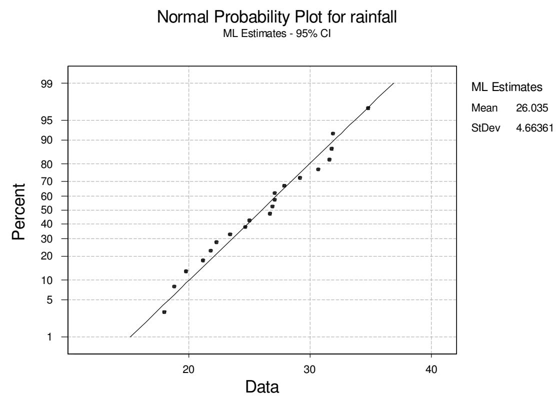
6) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.01,19} = 2.539$

7) $\bar{x} = 26.04$ $s = 4.78$ $n = 20$

$$t_0 = \frac{26.04 - 25}{4.78/\sqrt{20}} = 0.97$$

8) Since $0.97 < 2.539$, do not reject the null hypothesis and conclude there is insufficient evidence to indicate that the true mean rainfall is greater than 25 acre-feet at $\alpha = 0.01$. The $0.10 < \text{P-value} < 0.25$.

b.) the data on the normal probability plot fall along the straight line. Therefore there is evidence that the data are normally distributed.



c.) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|27 - 25|}{4.78} = 0.42$

Using the OC curve, Chart VI h) for $\alpha = 0.01$, d = 0.42, and n = 20, we get $\beta \cong 0.7$ and power of 1−0.7 = 0.3.

d) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|27.5 - 25|}{4.78} = 0.52$

Using the OC curve, Chart VI h) for $\alpha = 0.05$, d = 0.42, and $\beta \cong 0.1$ (Power=0.9),
$n = 75$.

e) 95% two sided confidence interval

$$\bar{x} - t_{0.025,24}\left(\frac{s}{\sqrt{n}}\right) \le \mu$$

$$26.03 - 2.776\left(\frac{4.78}{\sqrt{20}}\right) \le \mu$$

$$23.06 \le \mu$$

Since the lower limit of the CI is less than 25, we conclude that there is insufficient evidence to indicate that the true mean rainfall is not greater than 25 acre-feet at α=0.01.

9-33 a. 1) The parameter of interest is the true mean sodium content, μ.

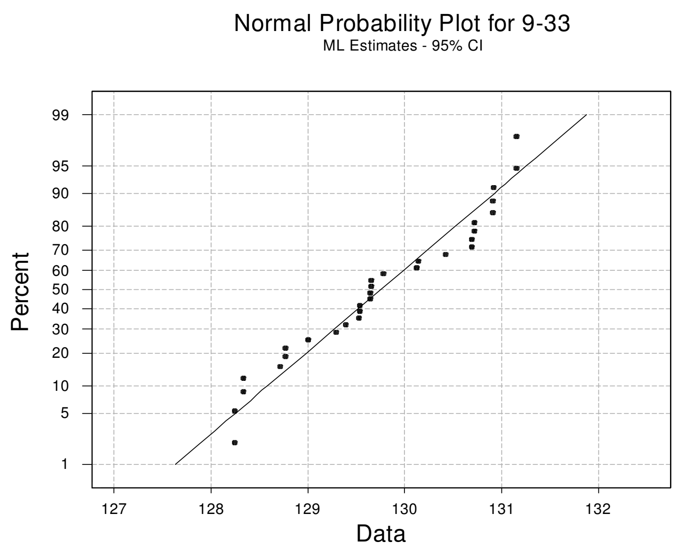
2) $H_0 : \mu = 130$
3) $H_1 : \mu \neq 130$
4) $\alpha = 0.05$
5) $t_0 = \dfrac{\overline{x} - \mu}{s / \sqrt{n}}$
6) Reject H_0 if $|t_0| > t_{\alpha/2,n-1}$ where $t_{\alpha/2,n-1} = 2.045$
7) $\overline{x} = 129.753$, s = 0.929 n=30

$$t_0 = \frac{129.753 - 130}{0.929\sqrt{30}} = -1.456$$

8) Since 1.456 < 2.064, do not reject the null hypothesis and conclude that the there is not sufficient evidence that the true mean sodium content is different from 130mg at $\alpha = 0.05$.

From table IV the t0 value is found between the values of 0.05 and 0.1 with 29 degrees of freedom, so 2*0.05<*P-value* = 2* 0.1 Therefore, 0.1< P-value<0.2.

b)



The assumption of normality appears to be reasonable.

c) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|130.5 - 130|}{0.929} = 0.538$

Using the OC curve, Chart VI e) for $\alpha = 0.05$, d = 0.53, and n = 30, we get $\beta \cong 0.2$ and power of 1−0.20 = 0.80.

d) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|130.1 - 130|}{0.929} = 0.11$

Using the OC curve, Chart VI e) for $\alpha = 0.05$, d = 0.11, and $\beta \cong 0.25$ (Power=0.75),
$n = 100$.

d) 95% two sided confidence interval

$$\bar{x} - t_{0.025,29}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,29}\left(\frac{s}{\sqrt{n}}\right)$$

$$129.753 - 2.045\left(\frac{0.929}{\sqrt{30}}\right) \le \mu \le 129.753 + 2.045\left(\frac{0.929}{\sqrt{30}}\right)$$

$$129.406 \le \mu \le 130.100$$

We can conclude that the mean sodium content is equal to 130 because that value is inside the confidence interval.

9-34 a.)1) The parameter of interest is the true mean tire life, μ.
2) $H_0 : \mu = 60000$
3) $H_1 : \mu > 60000$
4) $\alpha = 0.05$
5) $t_0 = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$
6) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.05,15} = 1.753$
7) $n = 16 \quad \bar{x} = 60{,}139.7 \quad s = 3645.94$

$$t_0 = \frac{60139.7 - 60000}{3645.94/\sqrt{16}} = 0.15$$

8) Since 0.15 < 1.753., do not reject the null hypothesis and conclude there is insufficient evidence to indicate that the true mean tire life is greater than 60,000 kilometers at $\alpha = 0.05$. The P-value > 0.40.

b.) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|61000 - 60000|}{3645.94} = 0.27$

Using the OC curve, Chart VI g) for $\alpha = 0.05$, d = 0.27, and $\beta \cong 0.1$ (Power=0.9),
$n = 4$.

Yes, the sample size of 16 was sufficient.

9-35. In order to use t statistics in hypothesis testing, we need to assume that the underlying distribution is normal.
1) The parameter of interest is the true mean impact strength, μ.
2) $H_0 : \mu = 1.0$
3) $H_1 : \mu > 1.0$
4) $\alpha = 0.05$
5) $t_0 = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$
6) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.05,19} = 1.729$
7) $\bar{x} = 1.25 \quad s = 0.25 \quad n = 20$

$$t_0 = \frac{1.25 - 1.0}{0.25/\sqrt{20}} = 4.47$$

8) Since 4.47 > 1.729, reject the null hypothesis and conclude there is sufficient evidence to indicate that the true mean impact strength is greater than 1.0 ft-lb/in at $\alpha = 0.05$. The P-value < 0.0005
.

9-36. In order to use t statistic in hypothesis testing, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true mean current, μ.
2) $H_0 : \mu = 300$
3) $H_1 : \mu > 300$
4) $\alpha = 0.05$
5) $t_0 = \dfrac{\overline{x} - \mu}{s / \sqrt{n}}$
6) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.05,9} = 1.833$
7) $n = 10 \quad \overline{x} = 317.2 \quad s = 15.7$

$$t_0 = \frac{317.2 - 300}{15.7/\sqrt{10}} = 3.46$$

8) Since 3.46 > 1.833, reject the null hypothesis and conclude there is sufficient evidence to indicate that the true mean current is greater than 300 microamps at $\alpha = 0.05$. The $0.0025 < P\text{-value} < 0.005$

9-37. a.) In order to use t statistics in hypothesis testing, we need to assume that the underlying distribution is normal.
1) The parameter of interest is the true mean coefficient of restitution, μ.
2) $H_0 : \mu = 0.635$
3) $H_1 : \mu > 0.635$
4) $\alpha = 0.05$
5) $t_0 = \dfrac{\overline{x} - \mu}{s / \sqrt{n}}$
6) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.05,39} = 1.685$
7) $\overline{x} = 0.624 \quad s = 0.013 \quad n = 40$

$$t_0 = \frac{0.624 - 0.635}{0.013/\sqrt{40}} = -5.35$$

8) Since –5.25 < 1.685, do not reject the null hypothesis and conclude that there is not sufficient evidence to indicate that the true mean coefficient of restitution is greater than 0.635 at $\alpha = 0.05$.

b.)The P-value > 0.4, based on Table IV. Minitab gives *P-value* = 1.

c) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|0.64 - 0.635|}{0.013} = 0.38$

Using the OC curve, Chart VI g) for $\alpha = 0.05$, d = 0.38, and n = 40, we get $\beta \cong 0.25$ and power of 1–0.25 = 0.75.

d) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|0.638 - 0.635|}{0.013} = 0.23$

Using the OC curve, Chart VI g) for $\alpha = 0.05$, d = 0.23, and $\beta \cong 0.25$ (Power=0.75),
$n = 40$.

9-38 a)In order to use t statistics in hypothesis testing, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true mean oxygen concentration, μ.

2) $H_0 : \mu = 4$

3) $H_1 : \mu \neq 4$

4) $\alpha = 0.01$

5) $t_0 = \dfrac{\overline{x} - \mu}{s / \sqrt{n}}$

6) Reject H_0 if $|t_0| > t_{a/2, n-1} = t_{0.005, 19} = 2.861$

7) $\overline{x} = 3.265$, s = 2.127, n = 20

$$t_0 = \frac{3.265 - 4}{2.127 / \sqrt{20}} = -1.55$$

8) Since -2.861<-1.55 <1.48, do not reject the null hypothesis and conclude that there is insufficient evidence to indicate that the true mean oxygen not equal 4 at $\alpha = 0.01$.

b.) The *P*-Value: 2*0.05<*P*-value<2*0.10 therefore 0.10< *P*-value<0.20

c.) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|3 - 4|}{2.127} = 0.47$

Using the OC curve, Chart VI f) for $\alpha = 0.01$, d = 0.47, and n = 20, we get $\beta \cong 0.70$ and power of 1–0.70 = 0.30.

d.) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|2.5 - 4|}{2.127} = 0.71$

Using the OC curve, Chart VI f) for $\alpha = 0.01$, d = 0.71, and $\beta \cong 0.10$ (Power=0.90), $n = 40$.

9-39 a)In order to use t statistics in hypothesis testing, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true mean cigar tar content, μ.

2) $H_0 : \mu = 1.5$

3) $H_1 : \mu > 1.5$

4) $\alpha = 0.05$

5) $t_0 = \dfrac{\overline{x} - \mu}{s / \sqrt{n}}$

6) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.05,29} = 1.699$

7) $\overline{x} = 1.529$ s = 0.0566 n = 30

$$t_0 = \frac{1.529 - 1.5}{0.0566 / \sqrt{30}} = 2.806$$

8) Since 2.806 > 1.699, reject the null hypothesis and conclude that there is sufficient evidence to indicate that the true mean tar content is greater than 1.5 at $\alpha = 0.05$.

b.) From table IV the t_0 value is found between the values of 0.0025 and 0.005 with 29 degrees of freedom. Therefore, 0.0025< P-value<0.005.

Minitab gives *P-value* = 0.004.

c) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|1.6 - 1.5|}{0.0566} = 1.77$

Using the OC curve, Chart VI g) for $\alpha = 0.05$, d = 1.77, and n = 30, we get $\beta \cong 0$ and power of 1–0 = 1.

e.) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|1.6 - 1.5|}{0.0566} = 1.77$

Using the OC curve, Chart VI g) for $\alpha = 0.05$, d = 1.77, and $\beta \cong 0.20$ (Power=0.80),
$n = 4$.

9-40 a) 1) The parameter of interest is the true mean height of female engineering students, μ.
2) $H_0 : \mu = 65$
3) $H_1 : \mu \neq 65$
4) $\alpha = 0.05$
5) $t_0 = \dfrac{\bar{x} - \mu}{s / \sqrt{n}}$
6) Reject H_0 if $|t_0| > t_{\alpha/2,n-1}$ where $t_{0.025,59} = 2.0281$
7) $\bar{x} = 65.811$ inches $s = 2.106$ inches n = 37

$$t_0 = \frac{65.811 - 65}{2.11/\sqrt{37}} = 2.34$$

8) Since 2.34 > 2.0281, reject the null hypothesis and conclude that there is sufficient evidence to indicate that the true mean height of female engineering students is not equal to 65 at $\alpha = 0.05$.

b.)P-value: 0.02<*P*-value<0.05.

c) $d = \dfrac{|62 - 65|}{2.11} = 1.42$, n=37 so, from the OC Chart VI e) for $\alpha = 0.05$, we find that $\beta \cong 0$. Therefore, the power $\cong$ 1.

d.) $d = \dfrac{|64 - 65|}{2.11} = 0.47$ for $\alpha = 0.05$, and $\beta \cong 0.2$ (Power=0.8).
$n^* = 40$.

9-41 a) In order to use t statistics in hypothesis testing, we need to assume that the underlying distribution is normal.
1) The parameter of interest is the true mean concentration of suspended solids, μ.
2) $H_0 : \mu = 55$
3) $H_1 : \mu \neq 55$
4) $\alpha = 0.05$
5) $t_0 = \dfrac{\bar{x} - \mu}{s / \sqrt{n}}$
6) Reject H_0 if $|t_0| > t_{\alpha/2,n-1}$ where $t_{0.025,59} = 2.000$
7) $\bar{x} = 59.87$ s = 12.50 n = 60

$$t_0 = \frac{59.87 - 55}{12.50/\sqrt{60}} = 3.018$$

8) Since 3.018 > 2.000, reject the null hypothesis and conclude that there is sufficient evidence to indicate that the true mean concentration of suspended solids is not equal to 55 at $\alpha = 0.05$.

b) From table IV the t0 value is found between the values of 0.001 and 0.0025 with 59 degrees of freedom, so 2*0.001<*P-value* = 2* 0.0025 Therefore, 0.002< P-value<0.005.
Minitab gives a p-value of 0.0038

c) $d = \frac{|50-55|}{12.50} = 0.4$, n=60 so, from the OC Chart VI e) for $\alpha = 0.05$, d= 0.4 and n=60 we find that $\beta \cong 0.2$. Therefore, the power = 1-0.2 = 0.8.

d) From the same OC chart, and for the specified power, we would need approximately 38 observations.

$d = \frac{|50-55|}{12.50} = 0.4$ Using the OC Chart VI e) for $\alpha = 0.05$, d = 0.4, and $\beta \cong 0.10$ (Power=0.90), $n = 75$.

9-42 a)In order to use t statistics in hypothesis testing, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true mean distance, μ.
2) $H_0 : \mu = 280$
3) $H_1 : \mu > 280$
4) $\alpha = 0.05$
5) $t_0 = \frac{\bar{x} - \mu}{s/\sqrt{n}}$
6) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.05,99} = 1.6604$
7) $\bar{x} = 260.3$ s = 13.41 n = 100

$$t_0 = \frac{260.3 - 280}{13.41/\sqrt{100}} = -14.69$$

8) Since –14.69 < 1.6604, do not reject the null hypothesis and conclude that there is insufficient evidence to indicate that the true mean distance is greater than 280 at $\alpha = 0.05$.

b.) From table IV the t_0 value is found above the value 0.005, therefore, the P-value is greater than 0.995.

c) $d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|290 - 280|}{13.41} = 0.75$

Using the OC curve, Chart VI g) for $\alpha = 0.05$, d = 0.75, and n = 100, we get $\beta \cong 0$ and power of 1–0 = 1.

f.) $d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|290 - 280|}{13.41} = 0.75$

Using the OC curve, Chart VI g) for $\alpha = 0.05$, d = 0.75, and $\beta \cong 0.20$ (Power=0.80), $n = 15$.

Section 9-4

9-43 a) In order to use the χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true standard deviation of the diameter, σ. However, the answer can be found by performing a hypothesis test on σ^2.

2) H_0 : $\sigma^2 = 0.0001$

3) H_1 : $\sigma^2 > 0.0001$

4) $\alpha = 0.01$

5) $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$

6) Reject H_0 if $\chi_0^2 > \chi^2_{\alpha,n-1}$ where $\chi^2_{0.01,14} = 29.14$

7) n = 15, $s^2 = 0.008$

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{14(0.008)^2}{0.0001} = 8.96$$

8) Since 8.96 < 29.14 do not reject H_0 and conclude there is insufficient evidence to indicate the true standard deviation of the diameter exceeds 0.01 at $\alpha = 0.01$.

b) P-value = $P(\chi^2 > 8.96)$ for 14 degrees of freedom: $0.5 < \text{P-value} < 0.9$

c) $\lambda = \dfrac{\sigma}{\sigma_0} = \dfrac{0.0125}{0.01} = 1.25$ power = 0.8, β=0.2

using chart VIk the required sample size is 50

9-44 a.) In order to use χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true variance of sugar content, σ^2. However, the answer can be found by performing a hypothesis test on σ^2.

2) H_0 : $\sigma^2 = 18$

3) H_1 : $\sigma^2 \neq 18$

4) $\alpha = 0.05$

5) $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$

6) Reject H_0 if $\chi_0^2 < \chi^2_{1-\alpha/2,n-1}$ where $\chi^2_{0.975,9} = 2.70$ or $\chi_0^2 > \chi^2_{\alpha,2,n-1}$ where $\chi^2_{0.025,9} = 19.02$

7) n = 10, s = 4.8

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{9(4.8)^2}{18} = 11.52$$

8) Since 11.52 < 19.02 do not reject H_0 and conclude there is insufficient evidence to indicate the true variance of sugar content is significantly different from 18 at $\alpha = 0.01$.

b.) *P*-value: The χ_0^2 is between 0.50 and 0.10. Therefore, $0.2 < P\text{-value} < 1$

c.) The 95% confidence interval includes the value 18, therefore, we could not be able to conclude that the variance was not equal to 18.

$$\frac{9(4.8)^2}{19.02} \le \sigma^2 \le \frac{9(4.8)^2}{2.70}$$

$$10.90 \le \sigma^2 \le 76.80$$

9-45 a) In order to use χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the standard deviation of tire life, σ. However, the answer can be found by performing a hypothesis test on σ^2.
2) H_0 : $\sigma^2 = 40{,}000$
3) H_1 : $\sigma^2 > 40{,}000$
4) $\alpha = 0.05$
5) $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$
6) Reject H_0 if $\chi_0^2 > \chi_{\alpha,n-1}^2$ where $\chi_{0.05,15}^2 = 25.00$
7) n = 16, $s^2 = (3645.94)^2$

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{15(3645.94)^2}{40000} = 4984.83$$

8) Since 4984.83 > 25.00 reject H_0 and conclude there is strong evidence to indicate the true standard deviation of tire life exceeds 200 km at $\alpha = 0.05$.

b) P-value = $P(\chi^2 > 4984.83)$ for 15 degrees of freedom P-value < 0.005

9-46 a) In order to use χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true standard deviation of Izod impact strength, σ. However, the answer can be found by performing a hypothesis test on σ^2.
2) H_0 : $\sigma^2 = (0.10)^2$
3) H_1 : $\sigma^2 \neq (0.10)^2$
4) $\alpha = 0.01$
5) $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$
6) Reject H_0 if $\chi_0^2 < \chi_{1-\alpha/2,n-1}^2$ where $\chi_{0.995,19}^2 = 6.84$ 27 or $\chi_0^2 > \chi_{\alpha,2,n-1}^2$ where $\chi_{0.005,19}^2 = 38.58$
7) n = 20, s = 0.25

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{19(0.25)^2}{(0.10)^2} = 118.75$$

8) Since 118.75 > 38.58 reject H_0 and conclude there is sufficient evidence to indicate the true standard deviation of Izod impact strength is significantly different from 0.10 at $\alpha = 0.01$.

b.) *P*-value: The P-value<0.005

c.) 99% confidence interval for σ:

First find the confidence interval for σ^2 :

For $\alpha = 0.01$ and n = 20, $\chi_{\alpha/2,n-1}^2 = \chi_{0.995,19}^2 = 6.84$ and $\chi_{1-\alpha/2,n-1}^2 = \chi_{0.005,19}^2 = 38.58$

$$\frac{19(0.25)^2}{38.58} \le \sigma^2 \le \frac{19(0.25)^2}{6.84}$$

$$0.03078 \le \sigma^2 \le 0.1736$$

Since 0.01 falls below the lower confidence bound we would conclude that the population standard deviation is not equal to 0.01.

9-47. a) In order to use χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true standard deviation of titanium percentage, σ. However, the answer can be found by performing a hypothesis test on σ^2.
2) $H_0: \sigma^2 = (0.25)^2$
3) $H_1: \sigma^2 \neq (0.25)^2$
4) $\alpha = 0.01$
5) $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$
6) Reject H_0 if $\chi_0^2 < \chi^2_{1-\alpha/2,n-1}$ where $\chi^2_{0.995,50} = 27.99$ or $\chi_0^2 > \chi^2_{\alpha,2,n-1}$ where $\chi^2_{0.005,50} = 79.49$
7) n = 51, s = 0.37

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{50(0.37)^2}{(0.25)^2} = 109.52$$

8) Since 109.52 > 79.49 we would reject H_0 and conclude there is sufficient evidence to indicate the true standard deviation of titanium percentage is significantly different from 0.25 at $\alpha = 0.01$.

b) 95% confidence interval for σ:
First find the confidence interval for σ^2 :
For $\alpha = 0.05$ and n = 51, $\chi^2_{\alpha/2,n-1} = \chi^2_{0.025,50} = 71.42$ and $\chi^2_{1-\alpha/2,n-1} = \chi^2_{0.975,50} = 32.36$

$$\frac{50(0.37)^2}{71.42} \leq \sigma^2 \leq \frac{50(0.37)^2}{32.36}$$
$$0.096 \leq \sigma^2 \leq 0.2115$$

Taking the square root of the endpoints of this interval we obtain,

$$0.31 < \sigma < 0.46$$

Since 0.25 falls below the lower confidence bound we would conclude that the population standard deviation is not equal to 0.25.

9-48 Using the chart in the Appendix, with $\lambda = \sqrt{\dfrac{0.012}{0.008}} = 1.22$ and n = 15 we find $\beta = 0.80$.

9-49 Using the chart in the Appendix, with $\lambda = \sqrt{\dfrac{40}{18}} = 1.49$ and $\beta = 0.10$, we find n = 30.

Section 9-5

9-50 a) The parameter of interest is the true proportion of engine crankshaft bearings exhibiting surface roughness.

2) $H_0 : p = 0.10$

3) $H_1 : p > 0.10$

4) $\alpha = 0.05$

5) $z_0 = \dfrac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion

6) Reject H_0 if $z_0 > z_\alpha$ where $z_\alpha = z_{0.05} = 1.65$

7) $x = 10$ $n = 85$ $\hat{p} = \dfrac{13}{300} = 0.043$

$$z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}} = \frac{10 - 85(0.10)}{\sqrt{85(0.10)(0.90)}} = 0.54$$

8) Since 0.53 < 1.65, do not reject the null hypothesis and conclude the true proportion of crankshaft bearings exhibiting surface roughness is not significantly greater than 0.10, at $\alpha = 0.05$.

9-51 $p = 0.15$, $p_0 = 0.10$, $n = 85$, and $z_{\alpha/2} = 1.96$

$$\beta = \Phi\left(\frac{p_0 - p + z_{\alpha/2}\sqrt{p_0(1-p_0)/n}}{\sqrt{p(1-p)/n}}\right) - \Phi\left(\frac{p_0 - p - z_{\alpha/2}\sqrt{p_0(1-p_0)/n}}{\sqrt{p(1-p)/n}}\right)$$

$$= \Phi\left(\frac{0.10 - 0.15 + 1.96\sqrt{0.10(1-0.10)/85}}{\sqrt{0.15(1-0.15)/85}}\right) - \Phi\left(\frac{0.10 - 0.15 - 1.96\sqrt{0.10(1-0.10)/85}}{\sqrt{0.15(1-0.15)/85}}\right)$$

$$= \Phi(0.36) - \Phi(-2.94) = 0.6406 - 0.0016 = 0.639$$

$$n = \left(\frac{z_{\alpha/2}\sqrt{p_0(1-p_0)} - z_\beta\sqrt{p(1-p)}}{p - p_0}\right)^2$$

$$= \left(\frac{1.96\sqrt{0.10(1-0.10)} - 1.28\sqrt{0.15(1-0.15)}}{0.15 - 0.10}\right)^2$$

$$= (10.85)^2 = 117.63 \cong 118$$

9-52 a) Using the information from Exercise 8-48, test

1) The parameter of interest is the true fraction defective integrated circuits

2) $H_0 : p = 0.05$

3) $H_1 : p \neq 0.05$

4) $\alpha = 0.05$

5) $z_0 = \dfrac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion

6) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $-z_{\alpha/2} = -z_{0.025} = -1.96$ or $z_0 > z_{\alpha/2}$ where $z_{\alpha/2} = z_{0.025} = 1.96$

7) $x = 13$ $n = 300$ $\hat{p} = \dfrac{13}{300} = 0.043$

$$z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}} = \frac{13 - 300(0.05)}{\sqrt{300(0.05)(0.95)}} = -0.53$$

8) Since $-0.53 > -1.65$, do not reject null hypothesis and conclude the true fraction of defective integrated circuits is not significantly less than 0.05, at $\alpha = 0.05$.

b.) The P-value: $2(1-\Phi(0.53))=2(1-0.70194)=0.59612$

9-53. a) Using the information from Exercise 8-48, test

1) The parameter of interest is the true fraction defective integrated circuits

2) $H_0 : p = 0.05$

3) $H_1 : p < 0.05$

4) $\alpha = 0.05$

5) $z_0 = \dfrac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion

6) Reject H_0 if $z_0 < -z_\alpha$ where $-z_\alpha = -z_{0.05} = -1.65$

7) $x = 13$ $n = 300$ $\hat{p} = \dfrac{13}{300} = 0.043$

$$z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}} = \frac{13 - 300(0.05)}{\sqrt{300(0.05)(0.95)}} = -0.53$$

8) Since $-0.53 > -1.65$, do not null hypothesis and conclude the true fraction of defective integrated circuits is not significantly less than 0.05, at $\alpha = 0.05$.

b) P-value $= 1 - \Phi(0.53) = 0.29806$

9-54 a) 1) The parameter of interest is the true proportion of engineering students planning graduate studies

2) $H_0 : p = 0.50$

3) $H_1 : p \neq 0.50$

4) $\alpha = 0.05$

5) $z_0 = \dfrac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion

6) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $-z_{\alpha/2} = -z_{0.025} = -1.96$ or $z_0 > z_{\alpha/2}$ where $z_{\alpha/2} = z_{0.025} = 1.96$

7) $x = 117$ $n = 484$ $\hat{p} = \dfrac{13}{300} = 0.043$

$$z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}} = \frac{117 - 484(0.5)}{\sqrt{484(0.5)(0.5)}} = -11.36$$

8) Since $-11.36 > -1.65$, reject the null hypothesis and conclude the true proportion of engineering students planning graduate studies is significantly different from 0.5, at $\alpha = 0.05$.

b.) P-value $= 2[1 - \Phi(11.36)] \cong 0$

c.) $\hat{p} = \dfrac{117}{484} = 0.242$

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq p \leq \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.242 - 1.96\sqrt{\frac{0.242(0.758)}{484}} \leq p \leq 0.242 - 1.96\sqrt{\frac{0.242(0.758)}{484}}$$

$$0.204 \leq p \leq 0.280$$

Since the 95% confidence interval does not contain the value 0.5, then conclude that the true proportion of engineering students planning graduate studies is significantly different from 0.5.

9-55. a) 1) The parameter of interest is the true percentage of polished lenses that contain surface defects, p.

2) $H_0 : p = 0.02$

3) $H_1 : p < 0.02$

4) $\alpha = 0.05$

5) $z_0 = \dfrac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion

6) Reject H_0 if $z_0 < -z_\alpha$ where $-z_\alpha = -z_{0.05} = -1.65$

7) $x = 6$ $n = 250$ $\hat{p} = \dfrac{6}{250} = 0.024$

$$z_0 = \frac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}} = \frac{0.024 - 0.02}{\sqrt{\dfrac{0.02(1-0.02)}{250}}} = 0.452$$

8) Since $0.452 > -1.65$ do not reject the null hypothesis and conclude the machine cannot be qualified at the 0.05 level of significance.

b) P-value $= \Phi(0.452) = 0.67364$

9-56 . a) 1) The parameter of interest is the true percentage of football helmets that contain flaws, p.

2) $H_0 : p = 0.1$

3) $H_1 : p > 0.1$

4) $\alpha = 0.01$

5) $z_0 = \dfrac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion

6) Reject H_0 if $z_0 > z_\alpha$ where $z_\alpha = z_{0.01} = 2.33$

7) x = 16 n = 200 $\hat{p} = \dfrac{16}{200} = 0.08$

$$z_0 = \frac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}} = \frac{0.08 - 0.10}{\sqrt{\dfrac{0.10(1-0.10)}{200}}} = -0.94$$

8) Since -0.452 < 2.33 do not reject the null hypothesis and conclude the proportion of football helmets with flaws does not exceed 10%.

b) P-value = $1-\Phi(0.94) = 1-.8264 = 0.67364$

9-57. The problem statement implies that H_0: p = 0.6, H_1: p > 0.6 and defines an acceptance region as

$\hat{p} \le \dfrac{315}{500} = 0.63$ and rejection region as $\hat{p} > 0.63$

a) The probability of a type 1 error is

$$\alpha = P(\hat{p} \ge 0.63 \mid p = 0.6) = P\left(Z \ge \frac{0.63 - 0.6}{\sqrt{\dfrac{0.6(0.4)}{500}}} \right) = P(Z \ge 1.37) = 1 - P(Z < 1.37) = 0.08535.$$

b) $\beta = P(\hat{P} \le 0.63 \mid p = 0.75) = P(Z \le -6.196) = 0.$

9-58 1) The parameter of interest is the true proportion of batteries that fail before 48 hours, p.

2) $H_0 : p = 0.002$

3) $H_1 : p < 0.002$

4) $\alpha = 0.01$

5) $z_0 = \dfrac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion

6) Reject H_0 if $z_0 < -z_\alpha$ where $-z_\alpha = -z_{0.01} = -2.33$

7) x = 15 n = 5000 $\hat{p} = \dfrac{15}{5000} = 0.003$

$$z_0 = \frac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}} = \frac{0.003 - 0.002}{\sqrt{\dfrac{0.002(1-0.998)}{5000}}} = 1.58$$

8) Since 1.58 > -2.33 do not reject the null hypothesis and conclude the proportion of proportion of cell phone batteries that fail is not less than 0.2% at α=0.01.

Section 9-7

9-59. Expected Frequency is found by using the Poisson distribution

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!} \text{ where } \lambda = [0(24) + 1(30) + 2(31) + 3(11) + 4(4)]/100 = 1.41$$

Value	0	1	2	3	4
Observed Frequency	24	30	31	11	4
Expected Frequency	30.12	36.14	21.69	8.67	2.60

Since value 4 has an expected frequency less than 3, combine this category with the previous category:

Value	0	1	2	3-4
Observed Frequency	24	30	31	15
Expected Frequency	30.12	36.14	21.69	11.67

The degrees of freedom are $k - p - 1 = 4 - 0 - 1 = 3$

a) 1) The variable of interest is the form of the distribution for X.
2) H_0: The form of the distribution is Poisson
3) H_1: The form of the distribution is not Poisson
4) $\alpha = 0.05$
5) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

6) Reject H_0 if $\chi_o^2 > \chi_{0.05,3}^2 = 7.81$

7) $$\chi_0^2 = \frac{(24-30.12)^2}{30.12} + \frac{(30-36.14)^2}{36.14} + \frac{(31-21.69)^2}{21.69} + \frac{(15-11.67)^2}{11.67} = 7.23$$

8) Since 7.23 < 7.81 do not reject H_0. We are unable to reject the null hypothesis that the distribution of X is Poisson.

b) The P-value is between 0.05 and 0.1 using Table III. P-value = 0.0649 (found using Minitab)

9-60. Expected Frequency is found by using the Poisson distribution

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!} \text{ where } \lambda = [1(1) + 2(11) + \ldots + 7(10) + 8(9)]/75 = 4.907$$

Estimated mean = 4.907

Value	1	2	3	4	5	6	7	8
Observed Frequency	1	11	8	13	11	12	10	9
Expected Frequency	2.7214	6.6770	10.9213	13.3977	13.1485	10.7533	7.5381	4.6237

Since the first category has an expected frequency less than 3, combine it with the next category:

Value	1-2	3	4	5	6	7	8
Observed Frequency	12	8	13	11	12	10	9
Expected Frequency	9.3984	10.9213	13.3977	13.1485	10.7533	7.5381	4.6237

The degrees of freedom are k − p − 1 = 7 − 1 − 1 = 5

a) 1) The variable of interest is the form of the distribution for the number of flaws.
2) H_0: The form of the distribution is Poisson
3) H_1: The form of the distribution is not Poisson
4) $\alpha = 0.01$
5) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

6) Reject H_0 if $\chi_o^2 > \chi_{0.01,5}^2 = 15.09$
7)

$$\chi_0^2 = \frac{(12 - 9.3984)^2}{9.3984} + \cdots + \frac{(9 - 4.6237)^2}{4.6237} = 6.955$$

8) Since 6.955 < 15.09 do not reject H_0. We are unable to reject the null hypothesis that the distribution of the number of flaws is Poisson.

b) P-value = 0.2237 (found using Minitab)

9-61. Estimated mean = 10.131

Value	5	6	8	9	10	11	12	13	14	15
Rel. Freq	0.067	0.067	0.100	0.133	0.200	0.133	0.133	0.067	0.033	0.067
Observed (Days)	2	2	3	4	6	4	4	2	1	2
Expected (Days)	1.0626	1.7942	3.2884	3.7016	3.7501	3.4538	2.9159	2.2724	1.6444	1.1106

Since there are several cells with expected frequencies less than 3, the revised table would be:

Value	5-8	9	10	11	12-15
Observed (Days)	7	4	6	4	9
Expected (Days)	6.1452	3.7016	3.7501	3.4538	7.9433

The degrees of freedom are k − p − 1 = 5 − 1 − 1 = 3

a) 1) The variable of interest is the form of the distribution for the number of calls arriving to a switchboard from noon to 1pm during business days.
2) H_0: The form of the distribution is Poisson
3) H_1: The form of the distribution is not Poisson
4) $\alpha = 0.05$
5) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

6) Reject H_0 if $\chi_o^2 > \chi_{0.05,3}^2 = 7.81$
7)

$$\chi_0^2 = \frac{(7-6.1452)^2}{6.1452} + \frac{(4-3.7016)^2}{3.7016} + \frac{(6-3.7501)^2}{3.7501} + \frac{(4-3.4538)^2}{3.4538} + \frac{(9-7.9433)^2}{7.9433} = 1.72$$

8) Since 1.72 < 7.81 do not reject H_0. We are unable to reject the null hypothesis that the distribution for the number of calls is Poisson.

b) The P-value is between 0.9 and 0.5 using Table III. P-value = 0.6325 (found using Minitab)

9-62 Use the binomial distribution to get the expected frequencies with the mean = np = 6(0.25) = 1.5

Value	0	1	2	3	4
Observed	4	21	10	13	2
Expected	8.8989	17.7979	14.8315	6.5918	1.6479

The expected frequency for value 4 is less than 3. Combine this cell with value 3:

Value	0	1	2	3-4
Observed	4	21	10	15
Expected	8.8989	17.7979	14.8315	8.2397

The degrees of freedom are $k - p - 1 = 4 - 0 - 1 = 3$

a) 1) The variable of interest is the form of the distribution for the random variable X.
2) H_0: The form of the distribution is binomial with n = 6 and p = 0.25
3) H_1: The form of the distribution is not binomial with n = 6 and p = 0.25
4) $\alpha = 0.05$
5) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{\left(O_i - E_i\right)^2}{E_i}$$

6) Reject H_0 if $\chi_o^2 > \chi_{0.05,3}^2 = 7.81$
7)

$$\chi_0^2 = \frac{(4-8.8989)^2}{8.8989} + \cdots + \frac{(15-8.2397)^2}{8.2397} = 10.39$$

8) Since 10.39 > 7.81 reject H_0. We can conclude that the distribution is not binomial with n = 6 and p = 0.25 at $\alpha = 0.05$.

b) P-value = 0.0155 (found using Minitab)

9-63 The value of p must be estimated. Let the estimate be denoted by $\hat{p}_{\text{sample}}$

$$\text{sample mean} = \frac{0(39)+1(23)+2(12)+3(1)}{75} = 0.6667$$

$$\hat{p}_{sample} = \frac{sample\ \ mean}{n} = \frac{0.6667}{24} = 0.02778$$

Value	0	1	2	3
Observed	39	23	12	1
Expected	38.1426	26.1571	8.5952	1.8010

Since value 3 has an expected frequency less than 3, combine this category with that of value 2:

Value	0	1	2-3
Observed	39	23	13
Expected	38.1426	26.1571	10.3962

The degrees of freedom are k − p − 1 = 3 − 1 − 1 = 1

a) 1) The variable of interest is the form of the distribution for the number of underfilled cartons, X.
2) H_0: The form of the distribution is binomial
3) H_1: The form of the distribution is not binomial
4) $\alpha = 0.05$
5) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{\left(O_i - E_i\right)^2}{E_i}$$

6) Reject H_0 if $\chi_o^2 > \chi_{0.05,1}^2 = 3.84$

7) $$\chi_0^2 = \frac{\left(39-38.1426\right)^2}{38.1426} + \frac{(23-26.1571)^2}{26.1571} + \frac{\left(13-10.3962\right)^2}{10.39} = 1.053$$

8) Since 1.053 < 3.84 do not reject H_0. We are unable to reject the null hypothesis that the distribution of the number of underfilled cartons is binomial at $\alpha = 0.05$.

b) The P-value is between 0.5 and 0.1 using Table III P-value = 0.3048 (found using Minitab)

9-64 Estimated mean = 49.6741 use Poisson distribution with λ=49.674
All expected frequencies are greater than 3.
The degrees of freedom are k − p − 1 = 26 − 1 − 1 = 24

a) 1) The variable of interest is the form of the distribution for the number of cars passing through the intersection.
2) H_0: The form of the distribution is Poisson
3) H_1: The form of the distribution is not Poisson
4) $\alpha = 0.05$
5) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{\left(O_i - E_i\right)^2}{E_i}$$

6) Reject H_0 if $\chi_o^2 > \chi_{0.05,24}^2 = 36.42$
7) Estimated mean = 49.6741

$$\chi_0^2 = 769.57$$

8) Since 769.57 >>> 36.42, reject H_0. We can conclude that the distribution is not Poisson at $\alpha = 0.05$.

b) P-value = 0 (found using Minitab)

Section 9-8

9-65. 1. The variable of interest is breakdowns among shift.
2. H_0: Breakdowns are independent of shift.
3. H_1: Breakdowns are not independent of shift.
4. $\alpha = 0.05$
5. The test statistic is:

$$\chi_0^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{(O_{ij}-E_{ij})^2}{E_{ij}}$$

6. The critical value is $\chi^2_{.05,6} = 12.592$
7. The calculated test statistic is $\chi^2_0 = 11.65$
8. $\chi_0^2 \not> \chi^2_{0.05,6}$, do not reject H_0 and conclude that the data provide insufficient evidence to claim that machine breakdown and shift are dependent at $\alpha = 0.05$.
P-value = 0.070 (using Minitab)

9-66 1. The variable of interest is calls by surgical-medical patients.
2. H_0:Calls by surgical-medical patients are independent of Medicare status.
3. H_1:Calls by surgical-medical patients are not independent of Medicare status.
4. $\alpha = 0.01$
5. The test statistic is:

$$\chi_0^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{(O_{ij}-E_{ij})^2}{E_{ij}}$$

6. The critical value is $\chi^2_{.01,1} = 6.637$
7. The calculated test statistic is $\chi^2_0 = 0.033$
8. $\chi_0^2 \not> \chi^2_{0.01,1}$, do not reject H_0 and conclude that the evidence is not sufficient to claim that surgical-medical patients and Medicare status are dependent. *P*-value = 0.85

9-67. 1. The variable of interest is statistics grades and OR grades.
2. H_0: Statistics grades are independent of OR grades.
3. H_1: Statistics and OR grades are not independent.
4. $\alpha = 0.01$
5. The test statistic is:

$$\chi_0^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{(O_{ij}-E_{ij})^2}{E_{ij}}$$

6. The critical value is $\chi^2_{.01,9} = 21.665$
7. The calculated critical value is $\chi^2_0 = 25.55$
8. $\chi_0^2 > \chi^2_{0.01,9}$ Therefore, reject H_0 and conclude that the grades are not independent at $\alpha = 0.01$.
P-value = 0.002

9-68 1. The variable of interest is characteristic among deflections and ranges.

2. H_0: Deflection and range are independent.
3. H_1: Deflection and range are not independent.
4. $\alpha = 0.05$
5. The test statistic is:

$$\chi_0^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{\left(O_{ij} - E_{ij}\right)^2}{E_{ij}}$$

6. The critical value is $\chi^2_{0.05,4} = 9.488$

7. The calculated test statistic is $\chi^2_0 = 2.46$

8. $\chi_0^2 \not> \chi^2_{0.05,4}$, do not reject H_0 and conclude that the evidence is not sufficient to claim that the data are not independent at $\alpha = 0.05$. *P*-value = 0.652

9-69. 1. The variable of interest is failures of an electronic component.
2. H_0: Type of failure is independent of mounting position.
3. H_1: Type of failure is not independent of mounting position.
4. $\alpha = 0.01$
5. The test statistic is:

$$\chi_0^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{\left(O_{ij} - E_{ij}\right)^2}{E_{ij}}$$

6. The critical value is $\chi^2_{0.01,3} = 11.344$

7. The calculated test statistic is $\chi^2_0 = 10.71$

8. $\chi_0^2 \not> \chi^2_{0.01,3}$, do not reject H_0 and conclude that the evidence is not sufficient to claim that the type of failure is not independent of the mounting position at $\alpha = 0.01$. *P*-value = 0.013

9-70 1. The variable of interest is opinion on core curriculum change.
2. H_0: Opinion of the change is independent of the class standing.
3. H_1: Opinion of the change is not independent of the class standing.
4. $\alpha = 0.05$
5. The test statistic is:

$$\chi_0^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{\left(O_{ij} - E_{ij}\right)^2}{E_{ij}}$$

6. The critical value is $\chi^2_{0.05,3} = 7.815$

7. The calculated test statistic is $\chi^2_{.0} = 26.97$.

8. $\chi_0^2 >>> \chi^2_{0.05,3}$, reject H_0 and conclude that the opinions on the change are not independent of class standing. *P*-value ≈ 0

Supplemental Exercises

9-71 Sample Mean = $\hat{p}$ Sample Variance = $\dfrac{\hat{p}(1-\hat{p})}{n}$

	Sample Size, n	Sampling Distribution	Sample Mean	Sample Variance
a.	50	Normal	p	$\dfrac{p(1-p)}{50}$
b.	80	Normal	p	$\dfrac{p(1-p)}{80}$
c.	100	Normal	p	$\dfrac{p(1-p)}{100}$

d) As the sample size increases, the variance of the sampling distribution decreases.

9-72

	n	Test statistic	P-value	conclusion
a.	50	$z_0 = \dfrac{0.095-0.10}{\sqrt{0.10(1-0.10)/50}} = -0.12$	0.4522	Do not reject H_0
b.	100	$z_0 = \dfrac{0.095-0.10}{\sqrt{0.10(1-0.10)/100}} = -0.15$	0.4404	Do not reject H_0
c.	500	$z_0 = \dfrac{0.095-0.10}{\sqrt{0.10(1-0.10)/500}} = -0.37$	0.3557	Do not reject H_0
d.	1000	$z_0 = \dfrac{0.095-0.10}{\sqrt{0.10(1-0.10)/1000}} = -0.53$	0.2981	Do not reject H_0

e. The P-value decreases as the sample size increases.

9-73. $\sigma = 12$, $\delta = 205 - 200 = 5$, $\dfrac{\alpha}{2} = 0.025$, $z_{0.025} = 1.96$,

a) n = 20: $\beta = \Phi\left(1.96 - \dfrac{5\sqrt{20}}{12}\right) = \Phi(0.163) = 0.564$

b) n = 50: $\beta = \Phi\left(1.96 - \dfrac{5\sqrt{50}}{12}\right) = \Phi(-0.986) = 1 - \Phi(0.986) = 1 - 0.839 = 0.161$

c) n = 100: $\beta = \Phi\left(1.96 - \dfrac{5\sqrt{100}}{12}\right) = \Phi(-2.207) = 1 - \Phi(2.207) = 1 - 0.9884 = 0.0116$

d) β, which is the probability of a Type II error, decreases as the sample size increases because the variance of the sample mean decreases. Consequently, the probability of observing a sample mean in the acceptance region centered about the incorrect value of 200 ml/h decreases with larger n.

9-74 $\sigma = 14$, $\delta = 205 - 200 = 5$, $\frac{\alpha}{2} = 0.025$, $z_{0.025} = 1.96$,

a) n = 20: $\beta = \Phi\left(1.96 - \frac{5\sqrt{20}}{14}\right) = \Phi(0.362) = 0.6406$

b) n = 50: $\beta = \Phi\left(1.96 - \frac{5\sqrt{50}}{14}\right) = \Phi(-0.565) = 1 - \Phi(0.565) = 1 - 0.7123 = 0.2877$

c) n = 100: $\beta = \Phi\left(1.96 - \frac{5\sqrt{100}}{14}\right) = \Phi(-1.611) = 1 - \Phi(1.611) = 1 - 0.9463 = 0.0537$

d) The probability of a Type II error increases with an increase in the standard deviation.

9-75. $\sigma = 8$, $\delta = 204 - 200 = -4$, $\frac{\alpha}{2} = 0.025$, $z_{0.025} = 1.96$.

a) n = 20: $\beta = \Phi\left(1.96 - \frac{4\sqrt{20}}{8}\right) = \Phi(-0.28) = 1 - \Phi(0.28) = 1 - 0.61026 = 0.38974$

Therefore, power = $1 - \beta = 0.61026$

b) n = 50: $\beta = \Phi\left(1.96 - \frac{4\sqrt{50}}{8}\right) = \Phi(-2.58) = 1 - \Phi(2.58) = 1 - 0.99506 = 0.00494$

Therefore, power = $1 - \beta = 0.995$

c) n = 100: $\beta = \Phi\left(1.96 - \frac{4\sqrt{100}}{8}\right) = \Phi(-3.04) = 1 - \Phi(3.04) = 1 - 0.99882 = 0.00118$

Therefore, power = $1 - \beta = 0.9988$

d) As sample size increases, and all other values are held constant, the power increases because the variance of the sample mean decreases. Consequently, the probability of a Type II error decreases, which implies the power increases.

9-76 $\alpha=0.01$

a.) n=25 $\beta = \Phi\left(z_{0.01} + \frac{85-86}{16/\sqrt{25}}\right) = \Phi(2.33 - 0.31) = \Phi(2.02) = 0.9783$

n=100 $\beta = \Phi\left(z_{0.01} + \frac{85-86}{16/\sqrt{100}}\right) = \Phi(2.33 - 0.63) = \Phi(1.70) = 0.9554$

n=400 $\beta = \Phi\left(z_{0.01} + \frac{85-86}{16/\sqrt{400}}\right) = \Phi(2.33 - 1.25) = \Phi(1.08) = 0.8599$

n=2500 $\beta = \Phi\left(z_{0.01} + \frac{85-86}{16/\sqrt{2500}}\right) = \Phi(2.33 - 3.13) = \Phi(-0.80) = 0.2119$

b.) n=25 $z_0 = \frac{86-85}{16/\sqrt{25}} = 0.31$ *P*-value: $1 - \Phi(0.31) = 1 - 0.6217 = 0.3783$

n=100 $z_0 = \dfrac{86-85}{16/\sqrt{100}} = 0.63$ *P*-value: $1-\Phi(0.63) = 1-0.7357 = 0.2643$

n=400 $z_0 = \dfrac{86-85}{16/\sqrt{400}} = 1.25$ *P*-value: $1-\Phi(1.25) = 1-0.8944 = 0.1056$

n=2500 $z_0 = \dfrac{86-85}{16/\sqrt{2500}} = 3.13$ *P*-value: $1-\Phi(3.13) = 1-0.9991 = 0.0009$

The data would be statistically significant when n=2500 at α=0.01

9-77. a) Rejecting a null hypothesis provides a *stronger conclusion* than failing to reject a null hypothesis. Therefore, place what we are trying to demonstrate in the alternative hypothesis.

Assume that the data follow a normal distribution.

b) 1) the parameter of interest is the mean weld strength, μ.
2) $H_0 : \mu = 150$
3) $H_1 : \mu > 150$
4) Not given
5) The test statistic is:

$$t_0 = \frac{\bar{x}-\mu_0}{s/\sqrt{n}}$$

6) Since no critical value is given, we will calculate the P-value
7) $\bar{x} = 153.7$, s= 11.3, n=20

$$t_0 = \frac{153.7-150}{11.3\sqrt{20}} = 1.46$$

P-value = $P(t \geq 1.46) = 0.05 < \text{p} - \text{value} < 0.10$

8) There is some modest evidence to support the claim that the weld strength exceeds 150 psi. If we used α = 0.01 or 0.05, we would not reject the null hypothesis, thus the claim would not be supported. If we used α = 0.10, we would reject the null in favor of the alternative and conclude the weld strength exceeds 150 psi.

9-78 a.) α=0.05

n=100 $\beta = \Phi\left(z_{0.05} + \dfrac{0.5-0.6}{\sqrt{0.5(0.5)/100}}\right) = \Phi(1.65-2.0) = \Phi(-0.35) = 0.3632$

$$Power = 1-\beta = 1-0.3632 = 0.6368$$

n=150 $\beta = \Phi\left(z_{0.05} + \dfrac{0.5-0.6}{\sqrt{0.5(0.5)/100}}\right) = \Phi(1.65-2.45) = \Phi(-0.8) = 0.2119$

$$Power = 1-\beta = 1-0.2119 = 0.7881$$

n=300 $\beta = \Phi\left(z_{0.05} + \dfrac{0.5-0.6}{\sqrt{0.5(0.5)/300}}\right) = \Phi(1.65-3.46) = \Phi(-1.81) = 0.03515$

$$Power = 1-\beta = 1-0.03515 = 0.96485$$

b.) α=0.01

n=100 $\beta = \Phi\left(z_{0.01} + \frac{0.5-0.6}{\sqrt{0.5(0.5)/100}} \right) = \Phi(2.33-2.0) = \Phi(0.33) = 0.6293$

$Power = 1-\beta = 1-0.6293 = 0.3707$

n=150 $\beta = \Phi\left(z_{0.01} + \frac{0.5-0.6}{\sqrt{0.5(0.5)/100}} \right) = \Phi(2.33-2.45) = \Phi(-0.12) = 0.4522$

$Power = 1-\beta = 1-0.4522 = 0.5478$

n=300 $\beta = \Phi\left(z_{0.01} + \frac{0.5-0.6}{\sqrt{0.5(0.5)/300}} \right) = \Phi(2.33-3.46) = \Phi(-1.13) = 0.1292$

$Power = 1-\beta = 1-0.1292 = 0.8702$

Decreasing the value of α decreases the power of the test for the different sample sizes.

c.) α=0.05

n=100 $\beta = \Phi\left(z_{0.05} + \frac{0.5-0.8}{\sqrt{0.5(0.5)/100}} \right) = \Phi(1.65-6.0) = \Phi(-4.35) \cong 0.0$

$Power = 1-\beta = 1-0 \cong 1$

The true value of *p* has a large effect on the power. The further *p* is away from p_0 the larger the power of the test.

d.)

$$n = \left(\frac{z_{\alpha/2}\sqrt{p_0(1-p_0)} - z_\beta\sqrt{p(1-p)}}{p-p_0} \right)^2$$

$$= \left(\frac{2.58\sqrt{0.5(1-0.50)} - 1.65\sqrt{0.6(1-0.6)}}{0.6-0.5} \right)^2 = (4.82)^2 = 23.2 \cong 24$$

$$n = \left(\frac{z_{\alpha/2}\sqrt{p_0(1-p_0)} - z_\beta\sqrt{p(1-p)}}{p-p_0} \right)^2$$

$$= \left(\frac{2.58\sqrt{0.5(1-0.50)} - 1.65\sqrt{0.8(1-0.8)}}{0.8-0.5} \right)^2 = (2.1)^2 = 4.41 \cong 5$$

The true value of *p* has a large effect on the power. The further *p* is away from p_0 the smaller the sample size that is required.

9-79 a) 1) the parameter of interest is the standard deviation, σ

2) $H_0 : \sigma^2 = 400$
3) $H_1 : \sigma^2 < 400$
4) Not given

5) The test statistic is: $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$

6) Since no critical value is given, we will calculate the p-value
7) n = 10, s = 15.7

$$\chi_0^2 = \frac{9(15.7)^2}{400} = 5.546$$

P-value = $P(\chi^2 < 5.546)$; $0.1 < P-\text{value} < 0.5$

8) The P-value is greater than any acceptable significance level, α, therefore we do not reject the null hypothesis. There is insufficient evidence to support the claim that the standard deviation is less than 20 microamps.

b) 7) n = 51, s = 20

$$\chi_0^2 = \frac{50(15.7)^2}{400} = 30.81$$

P-value = $P(\chi^2 < 30.81)$; $0.01 < P-\text{value} < 0.025$

8) The P-value is less than 0.05, therefore we reject the null hypothesis and conclude that the standard deviation is significantly less than 20 microamps.

c) Increasing the sample size increases the test statistic χ_0^2 and therefore decreases the P-value, providing more evidence against the null hypothesis.

9-80 a) 1) the parameter of interest is the variance of fatty acid measurements, σ^2
2) $H_0 : \sigma^2 = 1.0$
3) $H_1 : \sigma^2 \neq 1.0$
4) α=0.01

5) The test statistic is: $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$

6) $\chi^2_{0.995,5} = 0.41$ reject H_0 if $\chi_0^2 < 0.41$ or $\chi^2_{0.005,5} = 16.75$ reject H_0 if $\chi_0^2 > 16.75$

7) n = 6, s = 0.319

$$\chi_0^2 = \frac{5(0.319)^2}{1^2} = 0.509$$

P-value = $P(\chi^2 < 0.509)$; $0.01 < P-value < 0.02$

8) Since 0.509>0.41, do not reject the null hypothesis and conclude that there is insufficient evidence to conclude that the variance is not equal to 1.0. The *P*-value is greater than any acceptable significance level, α, therefore we do not reject the null hypothesis.

b) 1) the parameter of interest is the variance of fatty acid measurements, σ^2 (now n=51)
2) $H_0 : \sigma^2 = 1.0$

3) $H_1 : \sigma^2 \neq 1.0$
4) α=0.01

5) The test statistic is: $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$

6) $\chi^2_{0.995,50} = 27.99$ reject H_0 if $\chi_0^2 < 27.99$ or $\chi^2_{0.005,5} = 79.49$ reject H_0 if $\chi_0^2 > 79.49$

7) n = 51, s = 0.319

$$\chi_0^2 = \frac{50(0.319)^2}{1^2} = 5.09$$

P-value = $P(\chi^2 < 5.09)$; $\quad P - value < 0.01$

8) Since 5.09<27.99, reject the null hypothesis and conclude that there is sufficient evidence to conclude that the variance is not equal to 1.0. The *P*-value is smaller than any acceptable significance level, α, therefore we do reject the null hypothesis.

c.) The sample size changes the conclusion tha t is drawn. With a small sample size, the results are inconclusive. A larger sample size helps to make sure that the correct conclusion is drawn.

9-81. Assume the data follow a normal distribution.

a) 1) The parameter of interest is the standard deviation, σ.
2) $H_0 : \sigma^2 = (0.00002)^2$
3) $H_1 : \sigma^2 < (0.00002)^2$
4) $\alpha = 0.01$

5) The test statistic is: $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$

6) $\chi^2_{0.99,7} = 1.24$ reject H_0 if $\chi_0^2 < 1.24$

7) s = 0.00001 and $\alpha = 0.01$

$$\chi_0^2 = \frac{7(0.00001)^2}{(0.00002)^2} = 1.75$$

1.75 > 1.24, do not reject the null hypothesis; that is, there is insufficient evidence to conclude the standard deviation is at most 0.00002 mm.

b) Although the sample standard deviation is less than the hypothesized value of 0.00002, it is *not significantly less* (when $\alpha = 0.01$) than 0.00002 to conclude the standard deviation is at most 0.00002 mm. The value of 0.00001 could have occurred as a result of sampling variation.

9-82 Assume the data follow a normal distribution.

1) The parameter of interest is the standard deviation of the concentration, σ.
2) $H_0 : \sigma^2 = 4^2$
3) $H_1 : \sigma^2 < 4^2$
4) not given

5) The test statistic is: $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$

6) will be determined based on the *P*-value
7) s = 0.004 and n = 10

$$\chi_0^2 = \frac{9(0.004)^2}{(4)^2} = 0.000009$$

P-value = $P(\chi^2 < 0.00009)$; $\quad P - value \cong 0.$

The P-value is approximately 0, therefore we reject the null hypothesis and conclude that the standard deviation of the concentration is less than 4 grams per liter.

9-83. Create a table for the number of nonconforming coil springs (value) and the observed number of times the number appeared. One possible table is:

Value	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Obs	0	0	0	1	4	3	4	6	4	3	0	3	3	2	1	1	0	2	1	2

The value of p must be estimated. Let the estimate be denoted by $\hat{p}_{sample}$

$$\text{sample mean} = \frac{0(0) + 1(0) + 2(0) + \cdots + 19(2)}{40} = 9.325$$

$$\hat{p}_{sample} = \frac{sample \quad mean}{n} = \frac{9.325}{50} = 0.1865$$

Value	Observed	Expected
0	0	0.00165
1	0	0.01889
2	0	0.10608
3	1	0.38911
4	4	1.04816
5	3	2.21073
6	4	3.80118
7	6	5.47765
8	4	6.74985
9	3	7.22141
10	0	6.78777
11	3	5.65869
12	3	4.21619
13	2	2.82541
14	1	1.71190
15	1	0.94191
16	0	0.47237
17	2	0.21659
18	1	0.09103
19	2	0.03515

Since several of the expected values are less than 3, some cells must be combined resulting in the following table:

Value	Observed	Expected
0-5	8	3.77462
6	4	3.80118
7	6	5.47765
8	4	6.74985
9	3	7.22141
10	0	6.78777
11	3	5.65869
12	3	4.21619
≥13	9	6.29436

The degrees of freedom are k − p − 1 = 9 − 1 − 1 = 7

a) 1) The variable of interest is the form of the distribution for the number of nonconforming coil springs.
   2) H_0: The form of the distribution is binomial
   3) H_1: The form of the distribution is not binomial

4) $\alpha = 0.05$
5) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

6) Reject H_0 if $\chi_0^2 > \chi_{0.05,7}^2 = 14.07$
7)

$$\chi_0^2 = \frac{(8 - 3.77462)^2}{3.77462} + \frac{(4 - 3.8.011)^2}{3.8011} + \cdots + \frac{(9 - 6.29436)^2}{6.29436} = 17.929$$

8) Since 17.929 > 14.07 reject H_0. We are able to conclude the distribution of nonconforming springs is not binomial at $\alpha = 0.05$.
b) P-value = 0.0123 (found using Minitab)

9-84 Create a table for the number of errors in a string of 1000 bits (value) and the observed number of times the number appeared. One possible table is:

Value	0	1	2	3	4	5
Obs	3	7	4	5	1	0

The value of p must be estimated. Let the estimate be denoted by $\hat{p}_{sample}$

$$\text{sample mean} = \frac{0(3) + 1(7) + 2(4) + 3(5) + 4(1) + 5(0)}{20} = 1.7$$

$$\hat{p}_{sample} = \frac{sample \quad mean}{n} = \frac{1.7}{1000} = 0.0017$$

Value	0	1	2	3	4	5
Observed	3	7	4	5	1	0
Expected	3.64839	6.21282	5.28460	2.99371	1.27067	0.43103

Since several of the expected values are less than 3, some cells must be combined resulting in the following table:

Value	0	1	2	≥3
Observed	3	7	4	6
Expected	3.64839	6.21282	5.28460	4.69541

The degrees of freedom are $k - p - 1 = 4 - 1 - 1 = 2$

a) 1) The variable of interest is the form of the distribution for the number of errors in a string of 1000 bits.
2) H_0: The form of the distribution is binomial
3) H_1: The form of the distribution is not binomial
4) $\alpha = 0.05$
5) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

6) Reject H_0 if $\chi_0^2 > \chi_{0.05,2}^2 = 5.99$
7)

$$\chi_0^2 = \frac{(3 - 3.64839)^2}{3.64839} + \cdots + \frac{(6 - 4.69541)^2}{4.69541} = 0.88971$$

8) Since 0.88971 < 9.49 do not reject H_0. We are unable to reject the null hypothesis that the distribution of the number of errors is binomial at $\alpha = 0.05$.
b) P-value = 0.6409 (found using Minitab)

9-85 We can divide the real line under a standard normal distribution into eight intervals with equal probability. These intervals are [0,.32), [0.32, 0.675), [0.675, 1.15), [1.15, ∞) and their negative counterparts. The probability for each interval is p = 1/8 = .125 so the expected cell frequencies are E = *np* = (100) (0.125) = 12.5. The table of ranges and their corresponding frequencies is completed as follows.

Interval	Obs. Frequency.	Exp. Frequency.
$x \le 5332.5$	1	12.5
$5332.5 < x \le 5357.5$	4	12.5
$5357.5 < x \le 5382.5$	7	12.5
$5382.5 < x \le 5407.5$	24	12.5
$5407.5 < x \le 5432.5$	30	12.5
$5432.5 < x \le 5457.5$	20	12.5
$5457.5 < x \le 5482.5$	15	12.5
$x \ge 5482.5$	5	12.5

The test statistic is:

$$\chi_0^2 = \frac{(1-12.5)^2}{12.5} + \frac{(4-12.5)^2}{12.5} + \cdots + \frac{(15-12.5)^2}{12.5} + \frac{(5-12.5)^2}{12.5} = 63.36$$

and we would reject if this value exceeds $\chi^2_{0.05,5} = 11.07$. Since $\chi_o^2 > \chi^2_{0.05,5}$, reject the hypothesis that the data are normally distributed

9-86 a) In order to use t statistics in hypothesis testing, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true mean concentration of suspended solids, μ.
2) $H_0 : \mu = 50$
3) $H_1 : \mu < 50$
4) $\alpha = 0.05$
5) Since n>>30 we can use the normal distribution

$$z_0 = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

6) Reject H_0 if $z_0 < -z_\alpha$ where $z_{0.05} = 1.65$
7) $\bar{x} = 59.87$ s = 12.50 n = 60

$$z_0 = \frac{59.87 - 50}{12.50/\sqrt{60}} = 6.12$$

8) Since 6.12>-1.65, do not reject the null hypothesis and conclude there is insufficient evidence to indicate that the true mean concentration of suspended solids is less than 50 ppm at α = 0.05.

b) The P-value = $\Phi(6.12) \cong 1$.

c.) We can divide the real line under a standard normal distribution into eight intervals with equal probability. These intervals are [0,.32), [0.32, 0.675), [0.675, 1.15), [1.15, ∞) and their negative counterparts. The probability for each interval is p = 1/8 = .125 so the expected cell frequencies are E = np = (60) (0.125) = 7.5. The table of ranges and their corresponding frequencies is completed as follows.

Interval	Obs. Frequency.	Exp. Frequency.
$x \le 45.509$		7.5
$45.50 < x \le 51.435$		7.5
$51.43 < x \le 55.877$		7.5
$55.87 < x \le 59.8711$		7.5
$59.87 < x \le 63.874$		7.5
$63.87 < x \le 68.319$		7.5
$68.31 < x \le 74.248$		7.5
$x \ge 74.24$	6	7.5

The test statistic is:

$$\chi^2{}_o = \frac{(9-7.5)^2}{7.5} + \frac{(5-7.5)^2}{7.5} + \cdots + \frac{(8-7.5)^2}{7.5} + \frac{(6-7.5)^2}{7.5} = 5.06$$

and we would reject if this value exceeds $\chi^2{}_{0.05,5} = 11.07$. Since it does not, we cannot reject the hypothesis that the data are normally distributed.

9-87 a) In order to use t statistics in hypothesis testing, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true mean overall distance for this brand of golf ball, μ.

2) $H_0 : \mu = 270$

3) $H_1 : \mu < 270$

4) $\alpha = 0.05$

5) Since n>>30 we can use the normal distribution

$$z_0 = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

6) Reject H_0 if $z_0 < - z_\alpha$ where $z_{0.05} = 1.65$

7) $\bar{x} = 1.25$ s = 0.25 n = 100

$$z_0 = \frac{260.30 - 270.0}{13.41/\sqrt{100}} = -7.23$$

8) Since –7.23<-1.65, reject the null hypothesis and conclude there is sufficient evidence to indicate that the true mean distance is less than 270 yds at $\alpha = 0.05$.

b) The P-value ≅ 0

c) We can divide the real line under a standard normal distribution into eight intervals with equal probability. These intervals are [0,.32), [0.32, 0.675), [0.675, 1.15), [1.15, ∞) and their negative counterparts. The probability for each interval is p = 1/8 = .125 so the expected cell frequencies are E = np = (100) (0.125) = 12.5. The table of ranges and their corresponding frequencies is completed as follows.

Interval	Obs. Frequency.	Exp. Frequency.
$x \le 244.88$	16	12.5
$244.88 < x \le 251.25$	6	12.5
$251.25 < x \le 256.01$	17	12.5
$256.01 < x \le 260.30$	9	12.5
$260.30 < x \le 264.59$	13	12.5
$264.59 < x \le 269.35$	8	12.5
$269.35 < x \le 275.72$	19	12.5
$x \ge 275.72$	12	12.5

The test statistic is:

$$\chi^2{}_o = \frac{(16-12.5)^2}{12.5} + \frac{(6-12.5)^2}{12.5} + \cdots + \frac{(19-12.5)^2}{12.5} + \frac{(12-12.5)^2}{12.5} = 12$$

and we would reject if this value exceeds $\chi^2{}_{0.05,5} = 11.07$. Since it does, we can reject the hypothesis that the data are normally distributed.

9-88 a) In order to use t statistics in hypothesis testing, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true mean coefficient of restitution, μ.
2) $H_0 : \mu = 0.635$
3) $H_1 : \mu > 0.635$
4) $\alpha = 0.01$
5) Since n>30 we can use the normal distribution

$$z_0 = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

6) Reject H_0 if $z_0 > z_\alpha$ where $z_{0.05} = 2.33$
7) $\bar{x} = 0.324$ s = 0.0131 n = 40

$$z_0 = \frac{0.624 - 0.635}{0.0131/\sqrt{40}} = -5.31$$

8) Since –5.31< 2.33, do not reject the null hypothesis and conclude there is insufficient evidence to indicate that the true mean coefficient of restitution is greater than 0.635 at α = 0.01.

b) The P-value $\Phi(5.31) \cong 1$..

c.) If the lower bound of the CI was above the value 0.635 then we could conclude that the mean coefficient of restitution was greater than 0.635.

9-89 a) In order to use t statistics in hypothesis testing, we need to assume that the underlying distribution is normal. Use the t-test to test the hypothesis that the true mean is 2.5 mg/L.

1) State the parameter of interest: The parameter of interest is the true mean dissolved oxygen level, μ.
2) State the null hypothesis $H_0 : \mu = 2.5$
3) State the alternative hypothesis $H_1 : \mu \neq 2.5$
4) Give the significance level $\alpha = 0.05$
5) Give the statistic

$$t_0 = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

6) Reject H_0 if $|t_0| < t_{\alpha/2,n-1}$
7) find the sample statistic $\bar{x} = 3.265$ $s = 2.127$ $n = 20$

and calculate the t-statistic $t_0 = \dfrac{\bar{x} - \mu}{s / \sqrt{n}}$

8) Draw your conclusion and find the P-value.

b) Assume the data are normally distributed.

1) The parameter of interest is the true mean dissolved oxygen level, μ.
2) $H_0 : \mu = 2.5$
3) $H_1 : \mu \neq 2.5$
4) $\alpha = 0.05$
5)Test statistic

$$t_0 = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

6) Reject H_0 if $|t_0| > t_{\alpha/2,n-1}$ where $t_{\alpha/2,n-1} = t_{0.025,19b} = 2.093$
7) $\bar{x} = 3.265$ $s = 2.127$ $n = 20$

$$t_0 = \frac{3.265 - 2.5}{2.127 / \sqrt{20}} = 1.608$$

8) Since 1.608 < 2.093, do not reject the null hypotheses and conclude that the true mean is not significantly different from 2.5 mg/L

c.) The value of 1.608 is found between the columns of 0.05 and 0.1 of table IV. Therefore the P-value is between 0.1 and 0.2. Minitab gives a value of 0.124

d.) The confidence interval found in exercise 8-81 b. agrees with the hypothesis test above. The value of 2.5 is within the 95% confidence limits. The confidence interval shows that the interval is quite wide due to the large sample standard deviation value.

$$\bar{x} - t_{0.025,19} \frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + t_{0.025,19} \frac{s}{\sqrt{n}}$$

$$3.265 - 2.093 \frac{2.127}{\sqrt{20}} \leq \mu \leq 3.265 + 2.093 \frac{2.127}{\sqrt{20}}$$

$$2.270 \leq \mu \leq 4.260$$

9-90 a) In order to use t statistics in hypothesis testing, we need to assume that the underlying distribution is normal.

$$d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|73 - 75|}{1} = 2$$

Using the OC curve for $\alpha = 0.05$, $d = 2$, and n = 10, we get $\beta \cong 0.0$ and power of $1-0.0 \cong 1$.

$$d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|72 - 75|}{1} = 3$$

Using the OC curve for $\alpha = 0.05$, $d = 3$, and n = 10, we get $\beta \cong 0.0$ and power of $1-0.0 \cong 1$.

b) $d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|73 - 75|}{1} = 2$

Using the OC curve, Chart VI e) for $\alpha = 0.05$, $d = 2$, and $\beta \cong 0.1$ (Power=0.9),

$n^* = 5$. Therefore, $n = \frac{n^* + 1}{2} = \frac{5+1}{2} = 3$

$$d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|72 - 75|}{1} = 3$$

Using the OC curve, Chart VI e) for $\alpha = 0.05$, $d = 3$, and $\beta \cong 0.1$ (Power=0.9),

$n^* = 3$. Therefore, $n = \frac{n^* + 1}{2} = \frac{3+1}{2} = 2$

c) $\sigma = 2$.

$$d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|73-75|}{2} = 1$$

Using the OC curve for $\alpha = 0.05$, $d = 1$, and n = 10, we get $\beta \cong 0.10$ and power of $1-0.10 \cong 0.90$.

$$d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|72-75|}{2} = 1.5$$

Using the OC curve for $\alpha = 0.05$, $d = 1.5$, and n = 10, we get $\beta \cong 0.04$ and power of $1-0.04 \cong 0.96$.

$$d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|73-75|}{2} = 1$$

Using the OC curve, Chart VI e) for $\alpha = 0.05$, $d = 1$, and $\beta \cong 0.1$ (Power=0.9),

$n^* = 10$. Therefore, $n = \frac{n^*+1}{2} = \frac{10+1}{2} = 5.5$ $n \cong 6$

$$d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|72-75|}{2} = 1.5$$

Using the OC curve, Chart VI e) for $\alpha = 0.05$, $d = 3$, and $\beta \cong 0.1$ (Power=0.9),

$n^* = 7$. Therefore, $n = \frac{n^*+1}{2} = \frac{7+1}{2} = 4$

Increasing the standard deviation lowers the power of the test and increases the sample size required to obtain a certain power.

Mind Expanding Exercises

9-91 The parameter of interest is the true,μ.

$H_0 : \mu = \mu_0$

$H_1\ \mu \neq \mu_0$

9-92 a.) Reject H_0 if $z_0 < -z_{\alpha-\varepsilon}$ or $z_0 > z_\varepsilon$

$$P\left(\frac{\overline{X}-\mu_0}{\sigma/\sqrt{n}} < -\frac{\overline{X}-\mu_0}{\sigma/\sqrt{n}} = |\mu = \mu_0\right) + P\left(\frac{\overline{X}-\mu_0}{\sigma/\sqrt{n}} > \frac{\overline{X}-\mu_0}{\sigma/\sqrt{n}} |\mu = \mu_0\right) \text{P}$$

$$P(z_0 < -z_{\alpha-\varepsilon}) + P(z_0 > z_\varepsilon) = \Phi(-z_{\alpha-\varepsilon}) + 1 - \Phi(z_\varepsilon)$$

$$= ((\alpha - \varepsilon)) + (1-(1-\varepsilon)) = \alpha$$

b.) $\beta = P(z_\varepsilon \le \overline{X} \le z_\varepsilon$ when $\mu_1 = \mu_0 + d$)

or $\beta = P(-z_{\alpha-\varepsilon} < Z_0 < z_\varepsilon | \mu_1 = \mu_0 + \delta)$

$$\beta = P\left(-z_{\alpha-\varepsilon} < \frac{\bar{x}-\mu_0}{\sqrt{\sigma^2/n}} < z_\varepsilon | \mu_1 = \mu_0 + \delta\right)$$

$$= P\left(-z_{\alpha-\varepsilon} - \frac{\delta}{\sqrt{\sigma^2/n}} < Z < z_\varepsilon - \frac{\delta}{\sqrt{\sigma^2/n}}\right)$$

$$= \Phi\left(z_\varepsilon - \frac{\delta}{\sqrt{\sigma^2/n}}\right) - \Phi\left(-z_{\alpha-\varepsilon} - \frac{\delta}{\sqrt{\sigma^2/n}}\right)$$

]

9-93 1) The parameter of interest is the true mean number of open circuits, λ.

2) $H_0 : \lambda = 2$

3) $H_1 : \lambda > 2$

4) $\alpha = 0.05$

5) Since n>30 we can use the normal distribution

$$z_0 = \frac{\overline{X} - \lambda}{\sqrt{\lambda / n}}$$

6) Reject H_0 if $z_0 > z_\alpha$ where $z_{0.05}$ =1.65

7) $\bar{x}$= 1038/500=2.076 n = 500

$$z_0 = \frac{2.076 - 2}{2/\sqrt{500}} = 0.85$$

8) Since 0.85< 1.65, do not reject the null hypothesis and conclude there is insufficient evidence to indicate that the true mean number of open circuits is greater than 2 at $\alpha = 0.01$

9-94 1) The parameter of interest is the true standard deviation of the golf ball distance, λ.

2) $H_0 : \sigma = 10$

3) $H_1 : \sigma < 10$

4) α=0.05

5) Since n>30 we can use the normal distribution

$$z_0 = \frac{S - \sigma_0}{\sqrt{\sigma_0^2 /(2n)}}$$

6) Reject H_0 if $z_0 < z_\alpha$ where $z_{0.05}$ =-1.65

7) s= 13.41 n = 100

$$z_0 = \frac{13.41 - 10}{\sqrt{10^2 /(200)}} = 4.82$$

8) Since 4.82 > -1.65, do not reject the null hypothesis and conclude there is insufficient evidence to indicate that the true standard deviation is less than 10 at $\alpha = 0.05$

9-95 95% percentile $\theta = \mu + 1.645\sigma$

using $z_0 = \dfrac{S - \sigma_0}{\sqrt{\sigma_0^2 /(2n)}}$

95% percentile: $\overline{X} + 1.645\sqrt{s^2 /(2n)}$

$$S.E.(\theta) = \sigma / \sqrt{n} = \frac{s}{\sqrt{2n}\sqrt{n}} = s / \sqrt{3n} =$$

9-96 1) The parameter of interest is the true standard deviation of the golf ball distance, λ.
2) $H_0 : \theta = 285$
3) $H_1 : \sigma > 285$
4) α=0.05
5) Since n>30 we can use the normal distribution

$$z_0 = \frac{\hat{\Theta} - \vartheta_0}{\sqrt{\sigma^2 /(3n)}}$$

6) Reject H_0 if $z_0 > z_\alpha$ where $z_{0.05}$ =1.65
7) $\hat{\Theta}$= 282.36 n = 100

$$z_0 = \frac{282.36 - 285}{\sqrt{10^2 /(300)}} = -4.57$$

8) Since -4.82 > 1.65, do not reject the null hypothesis and conclude there is insufficient evidence to indicate that the true 95% is greater than 285 at $\alpha = 0.05$

9-97 1) The parameter of interest is the true mean number of open circuits, λ.
2) $H_0 : \lambda = \lambda_0$
3) $H_1 : \lambda \neq \lambda_0$
4) $\alpha = 0.05$
5) test statistic

$$\chi_0^2 = \frac{2\lambda\sum_{i=1}^{n} X_i - \lambda_0}{\sqrt{2\lambda\sum_{i=1}^{n} X_i}}$$

6) Reject H_0 if $\chi_0^2 > \chi^2_{a/2,2n}$ or $\chi_0^2 < \chi^2_{1-a/2,2n}$

7) compute $2\lambda\sum_{i=1}^{n} X_i$ and plug into

$$\chi_0^2 = \frac{2\lambda\sum_{i=1}^{n} X_i - \lambda_0}{\sqrt{2\lambda\sum_{i=1}^{n} X_i}}$$

8) make conclusions
alternative hypotheses
1) $H_0 : \lambda = \lambda_0$
$H_1 : \lambda > \lambda_0$
Reject H_0 if $\chi_0^2 > \chi^2_{a,2n}$

2) $H_0 : \lambda = \lambda_0$
$H_1 : \lambda < \lambda_0$
Reject H_0 if $\chi_0^2 < \chi^2_{a,2n}$