CHAPTER 4

Section 4-2

4-1. a) $P(1 < X) = \int_1^\infty e^{-x}dx = (-e^{-x})\Big|_1^\infty = e^{-1} = 0.3679$

b) $P(1 < X < 2.5) = \int_1^{2.5} e^{-x}dx = (-e^{-x})\Big|_1^{2.5} = e^{-1} - e^{-2.5} = 0.2858$

c) $P(X = 3) = \int_3^3 e^{-x}dx = 0$

d) $P(X < 4) = \int_0^4 e^{-x}dx = (-e^{-x})\Big|_0^4 = 1 - e^{-4} = 0.9817$

e) $P(3 \le X) = \int_3^\infty e^{-x}dx = (-e^{-x})\Big|_3^\infty = e^{-3} = 0.0498$

4-2. a) $P(x < X) = \int_x^\infty e^{-x}dx = (-e^{-x})\Big|_x^\infty = e^{-x} = 0.10$.

Then, x = –ln(0.10) = 2.3

b) $P(X \le x) = \int_0^x e^{-x}dx = (-e^{-x})\Big|_0^x = 1 - e^{-x} = 0.10$.

Then, x = –ln(0.9) = 0.1054

4-3 a) $P(X < 4) = \int_3^4 \frac{x}{8}dx = \frac{x^2}{16}\Big|_3^4 = \frac{4^2 - 3^2}{16} = 0.4375$, because $f_X(x) = 0$ for x < 3.

b) , $P(X > 3.5) = \int_{3.5}^5 \frac{x}{8}dx = \frac{x^2}{16}\Big|_{3.5}^5 = \frac{5^2 - 3.5^2}{16} = 0.7969$ because $f_X(x) = 0$ for x > 5.

c) $P(4 < X < 5) = \int_4^5 \frac{x}{8}dx = \frac{x^2}{16}\Big|_4^5 = \frac{5^2 - 4^2}{16} = 0.5625$

d) $P(X < 4.5) = \int_3^{4.5} \frac{x}{8}dx = \frac{x^2}{16}\Big|_3^{4.5} = \frac{4.5^2 - 3^2}{16} = 0.7031$

e) $P(X > 4.5) + P(X < 3.5) = \int_{4.5}^5 \frac{x}{8}dx + \int_3^{3.5} \frac{x}{8}dx = \frac{x^2}{16}\Big|_{4.5}^5 + \frac{x^2}{16}\Big|_3^{3.5} = \frac{5^2 - 4.5^2}{16} + \frac{3.5^2 - 3^2}{16} = 0.5$.

4-4 a) $P(1 < X) = \int_4^{\infty} e^{-(x-4)}dx = -e^{-(x-4)}\Big|_4^{\infty} = 1$, because $f_X(x) = 0$ for x < 4. This can also be obtained from the fact that $f_X(x)$ is a probability density function for 4 < x.

b) $P(2 \le X \le 5) = \int_4^5 e^{-(x-4)}dx = -e^{-(x-4)}\Big|_4^5 = 1 - e^{-1} = 0.6321$

c) $P(5 < X) = 1 - P(X \le 5)$. From part b., $P(X \le 5) = 0.6321$. Therefore, $P(5 < X) = 0.3679$.

d) $P(8 < X < 12) = \int_8^{12} e^{-(x-4)}dx = -e^{-(x-4)}\Big|_8^{12} = e^{-4} - e^{-8} = 0.0180$

e) $P(X < x) = \int_4^x e^{-(x-4)}dx = -e^{-(x-4)}\Big|_4^x = 1 - e^{-(x-4)} = 0.90$.

Then, x = 4 – ln(0.10) = 6.303

4-5 a) $P(0 < X) = 0.5$, by symmetry.

b) $P(0.5 < X) = \int_{0.5}^1 1.5x^2 dx = 0.5x^3\Big|_{0.5}^1 = 0.5 - 0.0625 = 0.4375$

c) $P(-0.5 \le X \le 0.5) = \int_{-0.5}^{0.5} 1.5x^2 dx = 0.5x^3\Big|_{-0.5}^{0.5} = 0.125$

d) P(X < –2) = 0

e) P(X < 0 or X > –0.5) = 1

f) $P(x < X) = \int_x^1 1.5x^2 dx = 0.5x^3\Big|_x^1 = 0.5 - 0.5x^3 = 0.05$

Then, x = 0.9655

4-6. a) $P(X > 3000) = \int_{3000}^{\infty} \frac{e^{\frac{-x}{1000}}}{1000} dx = -e^{\frac{-x}{1000}}\Big|_{3000}^{\infty} = e^{-3} = 0.05$

b) $P(1000 < X < 2000) = \int_{1000}^{2000} \frac{e^{\frac{-x}{1000}}}{1000} dx = -e^{\frac{-x}{1000}}\Big|_{1000}^{2000} = e^{-1} - e^{-2} = 0.233$

c) $P(X < 1000) = \int_0^{1000} \frac{e^{\frac{-x}{1000}}}{1000} dx = -e^{\frac{-x}{1000}}\Big|_0^{1000} = 1 - e^{-1} = 0.6321$

d) $P(X < x) = \int_0^x \frac{e^{\frac{-x}{1000}}}{1000} dx = -e^{\frac{-x}{1000}}\Big|_0^x = 1 - e^{-x/1000} = 0.10$.

Then, $e^{-x/1000} = 0.9$, and x = –1000 ln 0.9 = 105.36.

4-7 a) $P(X > 50) = \int_{50}^{50.25} 2.0dx = 2x\Big|_{50}^{50.25} = 0.5$

b) $P(X > x) = 0.90 = \int_{x}^{50.25} 2.0dx = 2x\Big|_{x}^{50.25} = 100.5 - 2x$

Then, 2x = 99.6 and x = 49.8.

4-8. a) $P(X < 74.8) = \int_{74.6}^{74.8} 1.25dx = 1.25x\Big|_{74.6}^{74.8} = 0.25$

b) P(X < 74.8 or X > 75.2) = P(X < 74.8) + P(X > 75.2) because the two events are mutually exclusive. The result is 0.25 + 0.25 = 0.50.

c) $P(74.7 < X < 75.3) = \int_{74.7}^{75.3} 1.25dx = 1.25x\Big|_{74.7}^{75.3} = 1.25(0.6) = 0.750$

4-9 a) P(X < 2.25 or X > 2.75) = P(X < 2.25) + P(X > 2.75) because the two events are mutually exclusive. Then, P(X < 2.25) = 0 and

P(X > 2.75) = $\int_{2.75}^{2.8} 2dx = 2(0.05) = 0.10$.

b) If the probability density function is centered at 2.5 meters, then $f_X(x) = 2$ for 2.3 < x < 2.8 and all rods will meet specifications.

4-10. Because the integral $\int_{x_1}^{x_2} f(x)dx$ is not changed whether or not any of the endpoints x_1 and x_2 are included in the integral, all the probabilities listed are equal.

Section 4-3

4-11. a) P(X<2.8) = P(X ≤ 2.8) because X is a continuous random variable.
Then, P(X<2.8) =F(2.8)=0.2(2.8) = 0.56.

b) $P(X > 1.5) = 1 - P(X \le 1.5) = 1 - 0.2(1.5) = 0.7$

c) $P(X < -2) = F_X(-2) = 0$

d) $P(X > 6) = 1 - F_X(6) = 0$

4-12. a) $P(X < 1.8) = P(X \le 1.8) = F_X(1.8)$ because X is a continuous random variable.

Then, $F_X(1.8) = 0.25(1.8) + 0.5 = 0.95$

b) $P(X > -1.5) = 1 - P(X \le -1.5) = 1 - .125 = 0.875$

c) P(X < -2) = 0

d) $P(-1 < X < 1) = P(-1 < X \le 1) = F_X(1) - F_X(-1) = .75 - .25 = 0.50$

4-13. Now, $f(x) = e^{-x}$ for 0 < x and $F_X(x) = \int_0^x e^{-x}dx = -e^{-x}\Big|_0^x = 1 - e^{-x}$

for 0 < x. Then, $F_X(x) = \begin{cases} 0, x \le 0 \\ 1 - e^{-x}, x > 0 \end{cases}$

4-14. Now, $f(x) = x/8$ for 3 < x < 5 and $F_X(x) = \int_3^x \frac{x}{8}dx = \frac{x^2}{16}\Big|_3^x = \frac{x^2 - 9}{16}$

for 0 < x. Then, $F_X(x) = \begin{cases} 0, x < 3 \\ \frac{x^2 - 9}{16}, 3 \le x < 5 \\ 1, x \ge 5 \end{cases}$

4-15. Now, $f(x) = e^{-(4-x)}$ for 4 < x and $F_X(x) = \int_4^x e^{-(4-x)}dx = -e^{-(4-x)}\Big|_4^x = 1 - e^{-(4-x)}$

for 4 < x.

Then, $F_X(x) = \begin{cases} 0, & x \le 4 \\ 1 - e^{-(4-x)}, & x > 4 \end{cases}$

4-16. Now, $f(x) = \frac{e^{-x/1000}}{1000}$ for 0 < x and

$F_X(x) = 1/1000 \int_4^x e^{-x/1000}dx = -e^{-x/1000)}\Big|_4^x = 1 - e^{-x/1000}$

for 0 < x.

Then, $F_X(x) = \begin{cases} 0, & x \le 0 \\ 1 - e^{-x/1000}, & x > 0 \end{cases}$

P(X>3000) = 1- P(X $\le$ 3000) = 1- F(3000) = $e^{-3000/1000}$ = 0.5

4-17. Now, f(x) = 1.25 for 74.6 < x < 75.4 and $F(x) = \int_{74.6}^x 1.25dx = 1.25x - 93.25$

for 74.6 < x < 75.4. Then,

$$F(x) = \begin{cases} 0, & x < 74.6 \\ 1.25x - 93.25, & 74.6 \le x < 75.4 \\ 1, & 75.4 \le x \end{cases}$$

$P(X > 75) = 1 - P(X \le 75) = 1 - F(75) = 1 - 0.5 = 0.5$ because X is a continuous random variable.

4-18 $f(x) = 2e^{-2x}, \quad x > 0$

4-19. $f\ (x) = \begin{cases} 0.2, & 0 < x < 4 \\ 0.04, & 4 \le x < 9 \end{cases}$

4-20. $f_X(x) = \begin{cases} 0.25, & -2 < x < 1 \\ 0.5, & 1 \le x < 1.5 \end{cases}$

4-21. $F(x) = \int_0^x 0.5x dx = \frac{0.5x^2}{2}\Big|_0^x = 0.25x^2$ for 0 < x < 2. Then,

$$F(x) = \begin{cases} 0, & x < 0 \\ 0.25x^2, & 0 \le x < 2 \\ 1, & 2 \le x \end{cases}$$

Section 4-4

4-22. $E(X) = \int_0^4 0.25x dx = 0.25 \frac{x^2}{2}\Big|_0^4 = 2$

$$V(X) = \int_0^4 0.25(x-2)^2 dx = 0.25 \frac{(x-2)^3}{3}\Big|_0^4 = \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$$

4-23. $E(X) = \int_0^4 0.125x^2 dx = 0.125 \frac{x^3}{3}\Big|_0^4 = 2.6667$

$$V(X) = \int_0^4 0.125x(x - \tfrac{8}{3})^2 dx = 0.125 \int_0^4 (x^3 - \tfrac{16}{3}x^2 + \tfrac{64}{9}x) dx$$

$$= 0.125(\tfrac{x^4}{4} - \tfrac{16}{3}\tfrac{x^3}{3} + \tfrac{64}{9} \cdot \tfrac{1}{2}x^2)\Big|_0^4 = 0.88889$$

4-24. $E(X)=\int_{-1}^{1}1.5x^3dx = 1.5\frac{x^4}{4}\Big|_{-1}^{1} = 0$

$V(X)=\int_{-1}^{1}1.5x^3(x-0)^2dx = 1.5\int_{-1}^{1}x^4dx$

$= 1.5\frac{x^5}{5}\Big|_{-1}^{1} = 0.6$

4-25. $E(X)=\int_{3}^{5}x\frac{x}{8}dx = \frac{x^3}{24}\Big|_{3}^{5} = \frac{5^3-3^3}{24} = 4.083$

$V(X)=\int_{3}^{5}(x-4.083)^2\frac{x}{8}dx = \int_{3}^{5}\left(\frac{x^3}{8}-\frac{8.166x^2}{8}+\frac{16.6709x}{8}\right)dx$

$= \frac{1}{8}\left(\frac{x^4}{4}-\frac{8.166x^3}{3}+\frac{16.6709x^2}{2}\right)\Big|_{3}^{5} = 0.3264$

4-26. $E(X)=\int_{49.75}^{50.25}2xdx = x^2\Big|_{49.75}^{50.25} = 50$

$V(X)=\int_{49.75}^{50.25}2(x-50)^2dx = 2\int_{49.75}^{50.25}(x^2-100x+2500)dx$

$= 2(\frac{x^3}{3}-100\frac{x^2}{2}+2500x)\Big|_{49.75}^{50.25}$

$= 0.0208$

4-27. a.) $E(X)=\int_{100}^{120}x\frac{600}{x^2}dx = 600\ln x\Big|_{100}^{120} = 109.39$

$V(X)=\int_{100}^{120}(x-109.39)^2\frac{600}{x^2}dx = 600\int_{100}^{120}1-\frac{2(109.39)}{x}+\frac{(109.39)^2}{x^2}dx$

$= 600(x-218.78\ln x-109.39^2x^{-1})\Big|_{100}^{120} = 33.19$

b.) Average cost per part = \$0.50*109.39 = \$54.70

4-28. $E(X)=\int_{1}^{\infty}x2x^{-3}dx = -2x^{-1}\Big|_{1}^{\infty} = 2$

4-29. a) $E(X) = \int_5^{\infty} x10e^{-10(x-5)}dx$.

Using integration by parts with $u = x$ and $dv = 10e^{-10(x-5)}dx$, we obtain

$$E(X) = -xe^{-10(x-5)}\Big|_5^{\infty} + \int_5^{\infty} e^{-10(x-5)}dx = 5 - \frac{e^{-10(x-5)}}{10}\Bigg|_5^{\infty} = 5.1$$

Now, $V(X) = \int_5^{\infty}(x-5.1)^2 10e^{-10(x-5)}dx$. Using the integration by parts with

$u = (x - 5.1)^2$ and

$dv = 10e^{-10(x-5)}$, we obtain $V(X) = -(x-5.1)^2 e^{-10(x-5)}\Big|_5^{\infty} + 2\int_5^{\infty}(x-5.1)e^{-10(x-5)}dx$.

From the definition of E(X) the integral above is recognized to equal 0.

Therefore, $V(X) = (5-5.1)^2 = 0.01$.

b) $P(X > 5.1) = \int_{5.1}^{\infty} 10e^{-10(x-5)}dx = -e^{-10(x-5)}\Big|_{5.1}^{\infty} = e^{-10(5.1-5)} = 0.3679$

4-30. a)

$$E(X) = \int_{1200}^{1210} x0.1dx = 0.05x^2\Big|_{1200}^{1210} = 1205$$

$$V(X) = \int_{1200}^{1210}(x-1205)^2 0.1dx = 0.1\frac{(x-1205)^3}{3}\Bigg|_{1200}^{1210} = 8.333$$

$$\textit{Therefore,} \quad \sigma_x = \sqrt{V(X)} = 2.887$$

b) Clearly, centering the process at the center of the specifications results in the greatest proportion of cables within specifications.

$$P(1195 < X < 1205) = P(1200 < X < 1205) = \int_{1200}^{1205} 0.1dx = 0.1x\Big|_{1200}^{1205} = 0.5$$

Section 4-5

4-31. a) E(X) = (5.5+1.5)/2 = 3.5,

$$V(X) = \frac{(5.5-1.5)^2}{12} = 1.333, \ \ and \ \ \sigma_x = \sqrt{1.333} = 1.155.$$

b) $P(X < 2.5) = \int_{1.5}^{2.5} 0.25dx = 0.25x\Big|_{1.5}^{2.5} = 0.25$

4-32. a) E(X) = (-1+1)/2 = 0,

$$V(X) = \frac{(1-(-1))^2}{12} = 1/3, \ \ and \ \sigma_x = 0.577$$

b) $P(-x < X < x) = \int_{-x}^{x} \tfrac{1}{2} dt = 0.5t\Big|_{-x}^{x} = 0.5(2x) = x$

Therefore, x should equal 0.90.

4-33. a) $f(x)$= 2.0 for 49.75 < x < 50.25.
E(X) = (50.25 + 49.75)/2 = 50.0,

$$V(X) = \frac{(50.25-49.75)^2}{12} = 0.0208, \ \ and \ \sigma_x = 0.144.$$

b) $F(x) = \int_{49.75}^{x} 2.0dx$ for 49.75 < x < 50.25. Therefore,

$$F(x) = \begin{cases} 0, & x < 49.75 \\ 2x - 99.5, & 49.75 \le x < 50.25 \\ 1, & 50.25 \le x \end{cases}$$

c) $P(X < 50.1) = F(50.1) = 2(50.1) - 99.5 = 0.7$

4-34. a) The distribution of X is $f(x)$ = 10 for 0.95 < x < 1.05. Now,

$$F_X(x) = \begin{cases} 0, & x < 0.95 \\ 10x - 9.5, & 0.95 \le x < 1.05 \\ 1, & 1.05 \le x \end{cases}$$

b) $P(X > 1.02) = 1 - P(X \le 1.02) = 1 - F_X(1.02) = 0.3$

c) If P(X > x)=0.90, then 1 – F(X) = 0.90 and F(X) = 0.10. Therefore, 10x - 9.5 = 0.10 and x = 0.96.

d) E(X) = (1.05 + 0.95)/2 = 1.00 and V(X) = $\frac{(1.05-0.95)^2}{12} = 0.00083$

4-35 $E(X) = \frac{(1.5+2.2)}{2} = 1.85\,\text{min}$

$$V(X) = \frac{(2.2-1.5)^2}{12} = 0.0408\,\text{min}^2$$

b) $P(X < 2) = \int_{1.5}^{2} \frac{1}{(2.2-1.5)}dx = \int_{1.5}^{2} (1/0.7)dx = (1/0.7)x\Big|_{1.5}^{2} = (1/0.7)(0.5) = 0.7143$

c.) $F(X) = \int_{1.5}^{x} \frac{1}{(2.2-1.5)}dx = \int_{1.5}^{x} (1/0.7)dx = (1/0.7)x\Big|_{1.5}^{x}$ for 1.5 < x < 2.2. Therefore,

$$F(x) = \begin{cases} 0, & x < 1.5 \\ (1/0.7)x - 2.14, & 1.5 \le x < 2.2 \\ 1, & 2.2 \le x \end{cases}$$

4-36 $f(x) = 0.04$ for 50< x <75

a) $P(X > 70) = \int_{70}^{75} 0.04dx = 0.2x\Big|_{70}^{75} = 0.2$

b) $P(X < 60) = \int_{50}^{60} 0.04dx = 0.04x\Big|_{50}^{60} = 0.4$

c) $E(X) = \frac{75+50}{2} = 62.5$ seconds

$$V(X) = \frac{(75-50)^2}{12} = 52.0833 \text{ seconds}^2$$

4-37. a) The distribution of X is $f(x)$ = 100 for 0.2050 < x < 0.2150. Therefore,

$$F(x) = \begin{cases} 0, & x < 0.2050 \\ 100x - 20.50, & 0.2050 \le x < 0.2150 \\ 1, & 0.2150 \le x \end{cases}$$

b) $P(X > 0.2125) = 1 - F(0.2125) = 1 - [100(0.2125) - 20.50] = 0.25$

c) If P(X > x)=0.10, then 1 – F(X) = 0.10 and F(X) = 0.90.
Therefore, 100x - 20.50 = 0.90 and x = 0.2140.

d) E(X) = (0.2050 + 0.2150)/2 = 0.2100 μm and

V(X) = $\frac{(0.2150-0.2050)^2}{12} = 8.33 \times 10^{-6}\,\mu\text{m}^2$

4-38. a) $P(X > 35) = \int_{35}^{40} 0.1dx = 0.1x\Big|_{35}^{40} = 0.5$

b) P(X > x)=0.90 and $P(X > x) = \int_{x}^{40} 0.1dt = 0.1(40 - x)$.

Now, 0.1(40-x) = 0.90 and x = 31

c) E(X) = (30 + 40)/2 = 35 and V(X) = $\dfrac{(40-30)^2}{12} = 8.33$

Section 4-6

4-39. a) P(Z<1.32) = 0.90658
b) P(Z<3.0) = 0.99865
c) P(Z>1.45) = 1 – 0.92647 = 0.07353
d) P(Z > –2.15) = p(Z < 2.15) = 0.98422
e) P(–2.34 < Z < 1.76) = P(Z<1.76) – P(Z > 2.34) = 0.95116

4-40. a) P(–1 < Z < 1) = P(Z < 1) – P(Z > 1)
= 0.84134 – (1 – 0.84134)
= 0.68268
b) P(–2 < Z < 2) = P(Z < 2) – [1 – P(Z < 2)]
= 0.9545
c) P(–3 < Z < 3) = P(Z < 3) – [1 – P(Z < 3)]
= 0.9973
d) P(Z > 3) = 1 – P(Z < 3)
= 0.00135
e) P(0 < Z < 1) = P(Z < 1) – P(Z < 0)
= 0.84134 – 0.5
= 0.34134

4-41 a) P(Z < 1.28) = 0.90
b) P(Z < 0) = 0.5
c) If P(Z > z) = 0.1, then P(Z < z) = 0.90 and z = 1.28
d) If P(Z > z) = 0.9, then P(Z < z) = 0.10 and z = –1.28
e) P(–1.24 < Z < z) = P(Z < z) – P(Z < –1.24)
= P(Z < z) – 0.10749.
Therefore, P(Z < z) = 0.8 + 0.10749 = 0.90749 and z = 1.33

4-42. a) Because of the symmetry of the normal distribution, the area in each tail of the distribution must equal 0.025. Therefore the value in Table II that corresponds to 0.975 is 1.96. Thus, z = 1.96.
b) Find the value in Table II corresponding to 0.995. z = 2.58.
c) Find the value in Table II corresponding to 0.84. z = 1.0
d) Find the value in Table II corresponding to 0.99865. z = 3.0.

4-43. a) P(X < 13) = P(Z < (13–10)/2)
= P(Z < 1.5)
= 0.93319

b) P(X > 9) = 1 – P(X < 9)
= 1 – P(Z < (9–10)/2)
= 1 – P(Z < –0.5)
= 0.69146.

c) P(6 < X < 14) = $P\left(\frac{6-10}{2} < Z < \frac{14-10}{2}\right)$
= P(–2 < Z < 2)
= P(Z < 2) –P(Z < – 2)]
= 0.9545.

d) P(2 < X < 4) = $P\left(\frac{2-10}{2} < Z < \frac{4-10}{2}\right)$
= P(–4 < Z < –3)
= P(Z < –3) – P(Z < –4)
= 0.00132

e) P(–2 < X < 8) = P(X < 8) – P(X < –2)
$= P\left(Z < \frac{8-10}{2}\right) - P\left(Z < \frac{-2-10}{2}\right)$
= P(Z < –1) – P(Z < –6)
= 0.15866.

4-44. a) P(X > x) = $P\left(Z > \frac{x-10}{2}\right)$ = 0.5. Therefore, $\frac{x-10}{2}$ = 0 and x = 10.

b) P(X > x) = $P\left(Z > \frac{x-10}{2}\right)$ = 1 – $P\left(Z < \frac{x-10}{2}\right)$
= 0.95.

Therefore, $P\left(Z < \frac{x-10}{2}\right)$ = 0.05 and $\frac{x-10}{2}$ = –1.64. Consequently, x = 6.72.

c) P(x < X < 10) = $P\left(\frac{x-10}{2} < Z < 0\right)$ = $P(Z < 0) - P\left(Z < \frac{x-10}{2}\right)$
= 0.5 – $P\left(Z < \frac{x-10}{2}\right)$ = 0.2.

Therefore, $P\left(Z < \frac{x-10}{2}\right)$ = 0.3 and $\frac{x-10}{2}$ = –0.52. Consequently, x = 8.96.

d) P(10 – x < X < 10 + x) = P(–x/2 < Z < x/2)= 0.95.
Therefore, x/2 = 1.96 and x = 3.92

e) P(10 – x < X < 10 + x) = P(–x/2 < Z < x/2) = 0.99.
Therefore, x/2 = 2.58 and x = 5.16

4-45. a) P(X < 11) = $P\left(Z < \frac{11-5}{4}\right)$

= P(Z < 1.5)
= 0.93319

b) P(X > 0) = $P\left(Z > \frac{0-5}{4}\right)$

= P(Z > –1.25)
= 1 – P(Z < –1.25)
= 0.89435

c) P(3 < X < 7) = $P\left(\frac{3-5}{4} < Z < \frac{7-5}{4}\right)$

= P(–0.5 < Z < 0.5)
= P(Z < 0.5) – P(Z < –0.5)
= 0.38292

d) P(–2 < X < 9) = $P\left(\frac{-2-5}{4} < Z < \frac{9-5}{4}\right)$

= P(–1.75 < Z < 1)
= P(Z < 1) – P(Z < –1.75)]
= 0.80128

e) P(2 < X < 8) = $P\left(\frac{2-5}{4} < Z < \frac{8-5}{4}\right)$

=P(–0.75 < Z < 0.75)
= P(Z < 0.75) – P(Z < –0.75)
= 0.54674

4-46. a) P(X > x) = $P\left(Z > \frac{x-5}{4}\right)$ = 0.5.

Therefore, x = 5.

b) P(X > x) = $P\left(Z > \frac{x-5}{4}\right)$ = 0.95.

Therefore, $P\left(Z < \frac{x-5}{4}\right)$ = 0.05

Therefore, $\frac{x-5}{4}$ = –1.64, and x = –1.56.

c) P(x < X < 9) = $P\left(\frac{x-5}{4} < Z < 1\right)$ = 0.2.

Therefore, P(Z < 1) – P(Z < $\frac{x-5}{4}$)= 0.2 where P(Z < 1) = 0.84134.

Thus P(Z < $\frac{x-5}{4}$) = 0.64134. Consequently, $\frac{x-5}{4}$ = 0.36 and x = 6.44.

d) P(3 < X < x) = $P\left(\frac{3-5}{4} < Z < \frac{x-5}{4}\right)$ = 0.95.

Therefore, $P\left(Z < \frac{x-5}{4}\right)$ – P(Z < –0.5) = 0.95 and $P\left(Z < \frac{x-5}{4}\right)$ – 0.30854 = 0.95.

Consequently,

$P\left(Z < \frac{x-5}{4}\right)$ = 1.25854. Because a probability can not be greater than one, there is no solution for x. In fact, P(3 < X) = P(–0.5 < Z) = 0.69146. Therefore, even if x is set to infinity the probability requested cannot equal 0.95.

e) P(5 – x < X <5 + x) = $P\left(\frac{5-x-5}{4} < Z < \frac{5+x-5}{4}\right)$

$= P\left(\frac{-x}{4} < Z < \frac{x}{4}\right)$ = 0.99

Therefore, x/4 = 2.58 and x = 10.32.

4-47. a) P(X < 6250) = $P\left(Z < \frac{6250-6000}{100}\right)$

= P(Z < 2.5)
= 0.99379

b) P(5800 < X < 5900) = $P\left(\frac{5800-6000}{100} < Z < \frac{5900-6000}{100}\right)$

=P(–2 < Z < –1)
= P(Z <– 1) – P(Z < –2)
= 0.13591

c) P(X > x) = $P\left(Z > \frac{x-6000}{100}\right)$ = 0.95.

Therefore, $\frac{x-6000}{100}$ = –1.65 and x = 5835.

4-48. a) P(X < 40) = $P\left(Z < \frac{40-35}{2}\right)$

= P(Z < 2.5)
= 0.99379

b) P(X < 30) = $P\left(Z < \frac{30-35}{2}\right)$

= P(Z < –2.5)
= 0.00621
0.621% are scrapped

4-49. a) P(X > 0.62) = $P\left(Z > \frac{0.62-0.5}{0.05}\right)$

= P(Z > 2.4)
= 1 – P(Z <2.4)
= 0.0082

b) P(0.47 < X < 0.63) = $P\left(\frac{0.47-0.5}{0.05} < Z < \frac{0.63-0.5}{0.05}\right)$

= P(–0.6 < Z < 2.6)
= P(Z < 2.6) – P(Z < –0.6)
= 0.99534 – 0.27425
= 0.72109

c) P(X < x) = $P\left(Z < \frac{x-0.5}{0.05}\right)$ = 0.90.

Therefore, $\frac{x-0.5}{0.05}$ = 1.28 and x = 0.564.

4-50. a) P(X < 12) = P(Z < $\frac{12-12.4}{0.1}$) = P(Z < –4) $\cong$ 0

b) P(X < 12.1) = $P\left(Z < \frac{12.1-12.4}{0.1}\right)$ = P(Z < –3) = 0.00135

and

P(X > 12.6) = $P\left(Z > \frac{12.6-12.4}{0.1}\right)$ = P(Z > 2) = 0.02275.

Therefore, the proportion of cans scrapped is 0.00135 + 0.02275 = 0.0241, or 2.41%

c) P(12.4 – x < X < 12.4 + x) = 0.99.

Therefore, $P\left(-\frac{x}{0.1} < Z < \frac{x}{0.1}\right)$ = 0.99

Consequently, $P\left(Z < \frac{x}{0.1}\right)$ = 0.995 and x = 0.1(2.58) = 0.258.

The limits are (12.142, 12.658).

4-51. a) P(X <45) = $P\left(Z < \frac{45-65}{5}\right)$ = P(Z < -3) = 0.00135

b) P(X > 65) = $P\left(Z > \frac{65-60}{5}\right)$ = P(Z >1) = 1- P(Z < 1)

= 1 - 0.841345= 0.158655

c) P(X < x) = $P\left(Z < \frac{x-60}{5}\right)$ = 0.99.

Therefore, $\frac{x-60}{5}$ = 2.33 and x = 72

4-52. a) If P(X > 12) = 0.999, then $P\left(Z > \frac{12-\mu}{0.1}\right)$ = 0.999.

Therefore, $\frac{12-\mu}{0.1}$ = –3.09 and μ = 12.309.

b) If P(X > 12) = 0.999, then $P\left(Z > \frac{12-\mu}{0.05}\right)$ = 0.999.

Therefore, $\frac{12-\mu}{0.05}$ = -3.09 and μ = 12.1545.

4-53. a) P(X > 0.5) = $P\left(Z > \frac{0.5-0.4}{0.05}\right)$

= P(Z > 2)
= 1 – 0.97725
= 0.02275

b) P(0.4 < X < 0.5) = $P\left(\frac{0.4-0.4}{0.05} < Z < \frac{0.5-0.4}{0.05}\right)$

= P(0 < Z < 2)
= P(Z < 2) – P(Z < 0)
= 0.47725

c) P(X > x) = 0.90, then $P\left(Z > \frac{x-0.4}{0.05}\right)$ = 0.90.

Therefore, $\frac{x-0.4}{0.05}$ = –1.28 and x = 0.336.

4-54 a) P(X > 70) = $P\left(Z > \frac{70-60}{4}\right)$

$= 1 - P(Z < 2.5)$

$= 1 - 0.99379 = 0.00621$

b) P(X < 58) = $P\left(Z < \frac{58-60}{4}\right)$

$= P(Z < -0.5)$

$= 0.308538$

c) 1,000,000 bytes * 8 bits/byte = 8,000,000 bits

$$\frac{8{,}000{,}000 \text{ bits}}{60{,}000 \text{ bits/sec}} = 133.33 \text{ seconds}$$

a) P(X > 90.3) + P(X < 89.7)

$$= P\left(Z > \frac{90.3 - 90.2}{0.1}\right) + P\left(Z < \frac{89.7 - 90.2}{0.1}\right)$$

= P(Z > 1) + P(Z < –5)

= 1 – P(Z < 1) + P(Z < –5)

=1 – 0.84134 +0

= 0.15866.

Therefore, the answer is 0.15866.

b) The process mean should be set at the center of the specifications; that is, at μ = 90.0.

c) P(89.7 < X < 90.3) = $P\left(\frac{89.7 - 90}{0.1} < Z < \frac{90.3 - 90}{0.1}\right)$

= P(–3 < Z < 3) = 0.9973.

The yield is 100*0.9973 = 99.73%

4-56. a) P(89.7 < X < 90.3) = $P\left(\frac{89.7 - 90}{0.1} < Z < \frac{90.3 - 90}{0.1}\right)$

= P(–3 < Z < 3)

= 0.9973.

P(X=10) = $(0.9973)^{10}$ = 0.9733

b) Let *Y* represent the number of cases out of the sample of 10 that are between 89.7 and 90.3 ml. Then Y follows a binomial distribution with *n*=10 and *p*=0.9973. Thus, E(*Y*)= 9.973 or 10.

4-57. a) P(50 < X < 80) = $P\left(\frac{50 - 100}{20} < Z < \frac{80 - 100}{20}\right)$

= P(–2.5 < Z < -1)

= P(Z < –1) – P(Z < –2.5)

= 0.15245.

b) P(X > x) = 0.10. Therefore, $P\left(Z > \frac{x - 100}{20}\right)$ = 0.10 and $\frac{x-100}{20}$ = 1.28.

Therefore, x = 126. hours

4-58. a) P(X < 5000) = $P\left(Z < \frac{5000-7000}{600}\right)$

= P(Z < –3.33) = 0.00043.

b) P(X > x) = 0.95. Therefore, $P\left(Z > \frac{x-7000}{600}\right)$ = 0.95 and $\frac{x-7000}{600}$ = –1.64.

Consequently, x = 6016.

c) P(X > 7000) = $P\left(Z > \frac{7000-7000}{600}\right) = P(Z > 0) = 0.5$

P(three lasers operating after 7000 hours) = $(1/2)^3$ =1/8

4-59. a) P(X > 0.0026) = $P\left(Z > \frac{0.0026-0.002}{0.0004}\right)$

= P(Z > 1.5)
= 1-P(Z < 1.5)
= 0.06681.

b) P(0.0014 < X < 0.0026) = $P\left(\frac{0.0014-0.002}{0.0004} < Z < \frac{0.0026-0.002}{0.0004}\right)$

= P(–1.5 < Z < 1.5)
= 0.86638.

c) P(0.0014 < X < 0.0026) = $P\left(\frac{0.0014-0.002}{\sigma} < Z < \frac{0.0026-0.002}{\sigma}\right)$

$= P\left(\frac{-0.0006}{\sigma} < Z < \frac{0.0006}{\sigma}\right)$.

Therefore, $P\left(Z < \frac{0.0006}{\sigma}\right)$ = 0.9975. Therefore, $\frac{0.0006}{\sigma}$ = 2.81 and σ = 0.000214.

4-60. a) P(X > 13) = $P\left(Z > \frac{13-12}{0.5}\right)$

= P(Z > 2)
= 0.02275

b) If P(X < 13) = 0.999, then $P\left(Z < \frac{13-12}{\sigma}\right)$ = 0.999.

Therefore, 1/ σ = 3.09 and σ = 1/3.09 = 0.324.

c) If P(X < 13) = 0.999, then $P\left(Z < \frac{13-\mu}{0.5}\right)$ = 0.999.

Therefore, $\frac{13-\mu}{0.5}$ = 3.09 and μ = 11.455

Section 4-7

4-61. a) E(X) = 200(0.4) = 80, V(X) = 200(0.4)(0.6) = 48 and $\sigma_X = \sqrt{48}$.

Then, $P(X \le 70) \cong P\left(Z \le \frac{70-80}{\sqrt{48}}\right) = P(Z \le -1.44) = 0.074934$

b)

$$P(70 < X \le 90) \cong P\left(\frac{70-80}{\sqrt{48}} < Z \le \frac{90-80}{\sqrt{48}}\right) = P(-1.44 < Z \le 1.44)$$
$$= 0.925066 - 0.074934 = 0.85013$$

4-62. a)

$$P(X < 4) = \binom{100}{0} 0.1^0 0.9^{100} + \binom{100}{1} 0.1^1 0.9^{99} + \binom{100}{2} 0.1^2 0.9^{98} + \binom{100}{3} 0.1^3 0.9^{97} = 0.0078$$

b) E(X) = 10, V(X) = 100(0.1)(0.9) = 9 and $\sigma_X = 3$.

Then, $P(X < 4) \cong P(Z < \frac{4-10}{3}) = P(Z < -2) = 0.023$

c) $P(8 < X < 12) \cong P(\frac{8-10}{3} < Z < \frac{12-10}{3}) = P(-0.67 < Z < 0.67) = 0.497$

4-63. Let X denote the number of defective chips in the lot.
Then, E(X) = 1000(0.02) = 20, V(X) = 1000(0.02)(0.98) = 19.6.

a) $P(X > 25) \cong P\left(Z > \frac{25-20}{\sqrt{19.6}}\right) = P(Z > 1.13) = 1 - P(Z \le 1.13) = 0.129$

b) $P(20 < X < 30) \cong P(0 < Z < \frac{10}{\sqrt{19.6}}) = P(0 < Z < 2.26)$
$= P(Z \le 2.26) - P(Z < 0) = 0.98809 - 0.5 = 0.488$

4-64. Let X denote the number of defective electrical connectors.
Then, E(X) = 25(100/1000) = 2.5, V(X) = 25(0.1)(0.9) = 2.25.

a) $P(X=0)=0.9^{25}=0.0718$

b) $P(X \le 0) \cong P(Z < \frac{0-2.5}{\sqrt{2.25}}) = P(Z < -1.67) = 0.047$

The approximation is smaller than the binomial. It is not satisfactory since $np<5$.

c) Then, E(X) = 25(100/500) = 5, V(X) = 25(0.2)(0.8) = 4.

$P(X=0)=0.8^{25}=0.00377$

$P(X \le 0) \cong P(Z < \frac{0-5}{\sqrt{4}}) = P(Z < -2.5) = 0.006$

Normal approximation is now closer to the binomial; however, it is still not satisfactory since $np = 5$ is not > 5.

4-65. Let X denote the number of original components that fail during the useful life of the product. Then, X is a binomial random variable with p = 0.001 and n = 5000. Also, E(X) = 5000 (0.001) = 5 and V(X) = 5000(0.001)(0.999) = 4.995.

$$P(X \geq 10) \cong P\left(Z \geq \frac{10-5}{\sqrt{4.995}}\right) = P(Z \geq 2.24) = 1 - P(Z < 2.24) = 1 - 0.987 = 0.013.$$

4-66. Let X denote the number of particles in 10 cm^2 of dust. Then, X is a Poisson random variable with $\lambda = 10(1000) = 10{,}000$. Also, E(X) = $\lambda = 10{,}000$ = V(X)

$$P(X > 10{,}000) \cong P(Z > \tfrac{10{,}000 - 10{,}000}{\sqrt{10{,}000}}) = P(Z > 0) = 0.5$$

4-67 Let X denote the number of errors on a web site. Then, X is a binomial random variable with p = 0.05 and n = 100. Also, E(X) = 100 (0.05) = 5 and V(X) = 100(0.05)(0.95) = 4.75

$$P(X \geq 1) \cong P\left(Z \geq \frac{1-5}{\sqrt{4.75}}\right) = P(Z \geq -1.84) = 1 - P(Z < -1.84) = 1 - 0.03288 = 0.96712$$

4-68. Let X denote the number of particles in 10 cm^2 of dust. Then, X is a Poisson random variable with $\lambda = 10{,}000$. Also, E(X) = $\lambda = 10{,}000$ =V(X)

a)

$$P(X \geq 20{,}000) \cong P(Z \geq \tfrac{20{,}000 - 10{,}000}{\sqrt{10{,}000}}) = P(Z \geq 100)$$

$$= 1 - P(Z < 100) = 1 - 1 = 0$$

b.) $P(X < 9{,}900) \cong P(Z < \tfrac{9{,}900 - 10{,}000}{\sqrt{10{,}000}}) = P(Z < -1) = 0.1587$

c.) If P($X > x$) = 0.01, then $P\left(Z > \frac{x - 10{,}000}{\sqrt{10{,}000}}\right) = 0.01$.

Therefore, $\frac{x - 10{,}000}{100} = 2.33$ and $x = 10{,}233$

4-69 Let X denote the number of hits to a web site. Then, X is a Poisson random variable with a of mean 10,000 per day. E(X) = λ = 10,000 and V(X) = 10,000

a)

$$P(X \geq 10{,}200) \cong P\left(Z \geq \frac{10{,}200 - 10{,}000}{\sqrt{10{,}000}}\right) = P(Z \geq 2) = 1 - P(Z < 2)$$
$$= 1 - 0.9772 = 0.0228$$

Expected value of hits days with more than 10,200 hits per day is (0.0228)*365=8.32 days per year

4-69 b.) Let Y denote the number of days per year with over 10,200 hits to a web site. Then, Y is a binomial random variable with n=365 and p=0.0228.
E(Y) = 8.32 and V(Y) = 365(0.0228)(0.9772)=8.13

$$P(Y > 15) \cong P\left(Z \geq \frac{15 - 8.32}{\sqrt{8.13}} \right) = P(Z \geq 2.34) = 1 - P(Z < 2.34)$$
$$= 1 - 0.9904 = 0.0096$$

4-70 E(X) = 1000(0.2) = 200 and V(X) = 1000(0.2)(0.8) = 160

a) $P(X \geq 225) \cong 1 - P(Z \leq \frac{225-200}{\sqrt{160}}) = 1 - P(Z \leq 1.98) = 1 - 0.97615 = 0.02385$

b) $P(175 \leq X \leq 225) \cong P(\frac{175-200}{\sqrt{160}} \leq Z \leq \frac{225-200}{\sqrt{160}}) = P(-1.98 \leq Z \leq 1.98)$
$= 0.97615 - 0.02385 = 0.9523$

c) If P($X > x$) = 0.01, then $P\left(Z > \frac{x - 200}{\sqrt{160}} \right) = 0.01$.

Therefore, $\frac{x-200}{\sqrt{160}} = 2.33$ and $x = 229.5$

4-71 X is the number of minor errors on a test pattern of 1000 pages of text. X is a Poisson random variable with a mean of 0.4 per page

a.) The number of errors per page is a random variable because it will be different for each page.

b.) $P(X = 0) = \frac{e^{-0.4} 0.4^0}{0!} = 0.670$

$P(X \geq 1) = 1 - P(X = 0) = 1 - 0.670 = 0.330$

The mean number of pages with one or more errors is 1000(0.330)=330 pages

c.) Let Y be the number of pages with errors.

$$P(Y > 350) \cong P\left(Z \geq \frac{350 - 330}{\sqrt{1000(0.330)(0.670)}} \right) = P(Z \geq 1.35) = 1 - P(Z < 1.35)$$
$$= 1 - 0.9115 = 0.0885$$

Section 4-9

4-72. a) $P(X \le 0) = \int_0^0 \lambda e^{-\lambda x} dx = 0$

b) $P(X \ge 2) = \int_2^\infty 2e^{-2x} dx = -e^{-2x}\Big|_2^\infty = e^{-4} = 0.0183$

c) $P(X \le 1) = \int_0^1 2e^{-2x} dx = -e^{-2x}\Big|_0^1 = 1 - e^{-2} = 0.8647$

d) $P(1 < X < 2) = \int_1^2 2e^{-2x} dx = -e^{-2x}\Big|_1^2 = e^{-2} - e^{-4} = 0.1170$

e) $P(X \le x) = \int_0^x 2e^{-2t} dt = -e^{-2t}\Big|_0^x = 1 - e^{-2x} = 0.05$ and x = 0.0256

4-73. If E(X) = 10, then $\lambda = 0.1$.

a) $P(X > 10) = \int_{10}^\infty 0.1e^{-0.1x} dx = -e^{-0.1x}\Big|_{10}^\infty = e^{-1} = 0.3679$

b) $P(X > 20) = -e^{-0.1x}\Big|_{20}^\infty = e^{-2} = 0.1353$

c) $P(X > 30) = -e^{-0.1x}\Big|_{30}^\infty = e^{-3} = 0.0498$

d) $P(X < x) = \int_0^x 0.1e^{-0.1t} dt = -e^{-0.1t}\Big|_0^x = 1 - e^{-0.1x} = 0.95$ and x = 29.96.

4-74. Let X denote the time until the first count. Then, X is an exponential random variable with $\lambda = 2$ counts per minute.

a) $P(X > 0.5) = \int_{0.5}^\infty 2e^{-2x} dx = -e^{-2x}\Big|_{0.5}^\infty = e^{-1} = 0.3679$

b) $P(X < \frac{10}{60}) = \int_0^{1/6} 2e^{-2x} dx = -e^{-2x}\Big|_0^{1/6} = 1 - e^{-1/3} = 0.2835$

c) $P(1 < X < 2) = -e^{-2x}\Big|_1^2 = e^{-2} - e^{-4} = 0.1170$

4-75. a) E(X) = $1/\lambda = 1/3 = 0.333$ minutes

b) V(X) = $1/\lambda^2 = 1/3^2 = 0.111$, $\sigma = 0.3333$

c) $P(X < x) = \int_0^x 3e^{-3t} dt = -e^{-3t}\Big|_0^x = 1 - e^{-3x} = 0.95$, x = 0.9986

4-76. The time to failure (in hours) for a laser in a cytometry machine is modeled by an exponential distribution with 0.00004.

a) $P(X > 20{,}000) = \int_{20000}^{\infty} 0.00004 e^{-.0.00004x} dx = -e^{-0.00004x}\Big|_{20000}^{\infty} = e^{-0.8} = 0.4493$

b) $P(X < 30{,}000) = \int_{30000}^{\infty} 0.00004 e^{-.0.00004x} dx = -e^{-0.00004x}\Big|_{0}^{30000} = 1 - e^{-1.2} = 0.6988$

c)

$$P(20{,}000 < X < 30{,}000) = \int_{20000}^{30000} 0.00004 e^{-.0.00004x} dx$$

$$= -e^{-0.00004x}\Big|_{20000}^{30000} = e^{-0.8} - e^{-1.2} = 0.1481$$

4-77. Let X denote the time until the first call. Then, X is exponential and $\lambda = \frac{1}{E(X)} = \frac{1}{15}$ calls/minute.

a) $P(X > 30) = \int_{30}^{\infty} \frac{1}{15} e^{-\frac{x}{15}} dx = -e^{-\frac{x}{15}}\Big|_{30}^{\infty} = e^{-2} = 0.1353$

b) The probability of at least one call in a 10-minute interval equals one minus the probability of zero calls in
a 10-minute interval and that is P(X > 10).

$P(X > 10) = -e^{-\frac{x}{15}}\Big|_{10}^{\infty} = e^{-2/3} = 0.5134$.

Therefore, the answer is 1- 0.5134 = 0.4866. Alternatively, the requested probability is equal to P(X < 10) = 0.4866.

c) $P(5 < X < 10) = -e^{-\frac{x}{15}}\Big|_{5}^{10} = e^{-1/3} - e^{-2/3} = 0.2031$

d) P(X < x) = 0.90 and $P(X < x) = -e^{-\frac{t}{15}}\Big|_{0}^{x} = 1 - e^{-x/15} = 0.90$. Therefore, x = 34.54 minutes.

4-78. Let X be the life of regulator. Then, X is an exponential random variable with $\lambda = 1 / E(X) = 1 / 6$

a) Because the Poisson process from which the exponential distribution is derived is memoryless, this probability is

$$P(X < 6) = \int_{0}^{6} \frac{1}{6} e^{-x/6} dx = -e^{-x/6}\Big|_{0}^{6} = 1 - e^{-1} = 0.6321$$

b) Because the failure times are memoryless, the mean time until the next failure is E(X) = 6 years.

4-79. Let X denote the time to failure (in hours) of fans in a personal computer. Then, X is an exponential random variable and $\lambda = 1/E(X) = 0.0003$.

a) P(X > 10,000) = $\int_{10,000}^{\infty} 0.0003 e^{-x0.0003} dx = -e^{-x0.0003} \Big|_{10,000}^{\infty} = e^{-3} = 0.0498$

b) P(X < 7,000) = $\int_{0}^{7,000} 0.0003 e^{-x0.0003} dx = -e^{-x0.0003} \Big|_{0}^{7,000} = 1 - e^{-2.1} = 0.8775$

4-80. Let X denote the time until a message is received. Then, X is an exponential random variable and $\lambda = 1/E(X) = 1/2$.

a) P(X > 2) = $\int_{2}^{\infty} \frac{1}{2} e^{-x/2} dx = -e^{-x/2} \Big|_{2}^{\infty} = e^{-1} = 0.3679$

b) The same as part a.

c) E(X) = 2 hours.

4-81. Let X denote the time until the arrival of a taxi. Then, X is an exponential random variable with $\lambda = 1/E(X) = 0.1$ arrivals/ minute.

a) P(X > 60) = $\int_{60}^{\infty} 0.1 e^{-0.1x} dx = -e^{-0.1x} \Big|_{60}^{\infty} = e^{-6} = 0.0025$

b) P(X < 10) = $\int_{0}^{10} 0.1 e^{-0.1x} dx = -e^{-0.1x} \Big|_{0}^{10} = 1 - e^{-1} = 0.6321$

4-82. a) P(X > x) = $\int_{x}^{\infty} 0.1 e^{-0.1t} dt = -e^{-0.1t} \Big|_{x}^{\infty} = e^{-0.1x} = 0.1$ and x = 23.03 minutes.

b) P(X < x) = 0.9 implies that P(X > x) = 0.1. Therefore, this answer is the same as part a.

c) P(X < x) = $-e^{-0.1t} \Big|_{0}^{x} = 1 - e^{-0.1x} = 0.5$ and x = 6.93 minutes.

4-83. Let X denote the distance between major cracks. Then, X is an exponential random variable with $\lambda = 1/E(X) = 0.2$ cracks/mile.

a) P(X > 10) = $\int_{10}^{\infty} 0.2 e^{-0.2x} dx = -e^{-0.2x} \Big|_{10}^{\infty} = e^{-2} = 0.1353$

b) Let Y denote the number of cracks in 10 miles of highway. Because the distance between cracks is exponential, Y is a Poisson random variable with $\lambda = 10(0.2) = 2$ cracks per 10 miles.

$$P(Y = 2) = \frac{e^{-2} 2^2}{2!} = 0.2707$$

c) $\sigma_X = 1/\lambda = 5$ miles.

4-84. a) $P(12 < X < 15) = \int_{12}^{15} 0.2e^{-0.2x}dx = -e^{-0.2x}\Big|_{12}^{15} = e^{-2.4} - e^{-3} = 0.0409$

b) P(X > 5) = $-e^{-0.2x}\Big|_{5}^{\infty} = e^{-1} = 0.3679$. By independence of the intervals in a Poisson process, the answer is $0.3679^2 = 0.1353$. Alternatively, the answer is P(X > 10) = $e^{-2} = 0.1353$. The probability does depend on whether or not the lengths of highway are consecutive.

c) By the memoryless property, this answer is P(X > 10) = 0.1353 from part b.

4-85. Let X denote the lifetime of an assembly. Then, X is an exponential random variable with $\lambda = 1/E(X) = 1/400$ failures per hour.

a) P(X < 100) = $\int_{0}^{100} \frac{1}{400}e^{-x/400}dx = -e^{-x/400}\Big|_{0}^{100} = 1 - e^{-0.25} = 0.2212$

b) P(X > 500) = $-e^{-x/400}\Big|_{500}^{\infty} = e^{-5/4} = 0.2865$

c) From the memoryless property of the exponential, this answer is the same as part a., P(X < 100) = 0.2212.

4-86. a) Let U denote the number of assemblies out of 10 that fail before 100 hours. By the memoryless property of a Poisson process, U has a binomial distribution with n = 10 and p =0.2212 (from Exercise 4-85a). Then,

$P(U \geq 1) = 1 - P(U = 0) = 1 - \binom{10}{0}0.2212^0(1-0.2212)^{10} = 0.9179$

b) Let V denote the number of assemblies out of 10 that fail before 800 hours. Then, V is a binomial random variable with n = 10 and p = P(X < 800), where X denotes the lifetime of an assembly.

Now, P(X < 800) = $\int_{0}^{800} \frac{1}{400}e^{-x/400}dx = -e^{-x/400}\Big|_{0}^{800} = 1 - e^{-2} = 0.8647$.

Therefore, P(V = 10) = $\binom{10}{10}0.8647^{10}(1-0.8647)^0 = 0.2337$.

4-87. Let X denote the number of calls in 3 hours. Because the time between calls is an exponential random variable, the number of calls in 3 hours is a Poisson random variable. Now, the mean time between calls is 0.5 hours and $\lambda = 1/0.5 = 2$ calls per hour = 6 calls in 3 hours.

$$P(X \geq 4) = 1 - P(X \leq 3) = 1 - \left[\frac{e^{-6}6^0}{0!} + \frac{e^{-6}6^1}{1!} + \frac{e^{-6}6^2}{2!} + \frac{e^{-6}6^3}{3!}\right] = 0.8488$$

4-88. Let Y denote the number of arrivals in one hour. If the time between arrivals is exponential, then the count of arrivals is a Poisson random variable and $\lambda = 1$ arrival per hour.

P(Y > 3) = $1 - P(Y \leq 3) = 1 - \left[\frac{e^{-1}1^0}{0!} + \frac{e^{-1}1^1}{1!} + \frac{e^{-1}1^2}{2!} + \frac{e^{-1}1^3}{3!}\right] = 0.01899$

4-89. a) From Exercise 4-88, P(Y > 3) = 0.01899. Let W denote the number of one-hour intervals out of 30 that contain more than 3 arrivals. By the memoryless property of a Poisson process, W is a binomial random variable with n = 30 and p = 0.01899.

P(W = 0) = $\binom{30}{0} 0.01899^0 (1-0.01899)^{30} = 0.5626$

b) Let X denote the time between arrivals. Then, X is an exponential random variable with $\lambda = 1$ arrivals per hour. P(X > x) = 0.1 and $P(X > x) = \int_x^\infty 1e^{-1t}dt = -e^{-1t}\Big|_x^\infty = e^{-1x} = 0.1$.

Therefore, x = 2.3 hours.

4-90. Let X denote the number of calls in 30 minutes. Because the time between calls is an exponential random variable, X is a Poisson random variable with $\lambda = 1/E(X) = 0.1$ calls per minute = 3 calls per 30 minutes.

a) P(X > 3) = $1 - P(X \le 3) = 1 - \left[\frac{e^{-3}3^0}{0!} + \frac{e^{-3}3^1}{1!} + \frac{e^{-3}3^2}{2!} + \frac{e^{-3}3^3}{3!}\right] = 0.3528$

b) P(X = 0) = $\frac{e^{-3}3^0}{0!} = 0.04979$

c) Let Y denote the time between calls in minutes. Then, $P(Y \ge x) = 0.01$ and

$P(Y \ge x) = \int_x^\infty 0.1e^{-0.1t}dt = -e^{-0.1t}\Big|_x^\infty = e^{-0.1x}$. Therefore, $e^{-0.1x} = 0.01$ and x = 46.05 minutes.

4-91. a) From Exercise 4-90, P(Y > 120) = $\int_{120}^\infty 0.1e^{-0.1y}dy = -e^{-0.1y}\Big|_{120}^\infty = e^{-12} = 6.14 \times 10^{-6}$.

b) Because the calls are a Poisson process, the number of calls in disjoint intervals are independent. From Exercise 4-90 part b., the probability of no calls in one-half hour is $e^{-3} = 0.04979$. Therefore, the answer is $\left[e^{-3}\right]^4 = e^{-12} = 6.14 \times 10^{-6}$. Alternatively, the answer is the probability of no calls in two hours. From part a. of this exercise, this is e^{-12}.

c) Because a Poisson process is memoryless, probabilities do not depend on whether or not intervals are consecutive. Therefore, parts a. and b. have the same answer.

4-92. a) $P(X > \theta) = \int_\theta^\infty \frac{1}{\theta}e^{-x/\theta}dx = -e^{-x/\theta}\Big|_\theta^\infty = e^{-1} = 0.3679$

b) $P(X > 2\theta) = -e^{-x/\theta}\Big|_{2\theta}^\infty = e^{-2} = 0.1353$

c) $P(X > 3\theta) = -e^{-x/\theta}\Big|_{3\theta}^\infty = e^{-3} = 0.0498$

d) The results do not depend on θ.

4-93. X is an exponential random variable with $\lambda = 0.2$ flaws per meter.

a) E(X) = $1/\lambda = 5$ meters.

b) P(X > 10) = $\int_{10}^{\infty} 0.2e^{-0.2x}dx = -e^{-0.2x}\Big|_{10}^{\infty} = e^{-2} = 0.1353$

c) No, see Exercise 4-91 part c.

d) P(X < x) = 0.90. Then, P(X < x) = $-e^{-0.2t}\Big|_{0}^{x} = 1 - e^{-0.2x}$. Therefore, $1 - e^{-0.2x} = 0.9$ and x = 11.51.

4-94. P(X > 8) = $\int_{8}^{\infty} 0.2e^{-0.2x}dx = -e^{-8/5} = 0.2019$

The distance between successive flaws is either less than 8 meters or not. The distances are independent and P(X > 8) = 0.2019. Let Y denote the number of flaws until the distance exceeds 8 meters. Then, Y is a geometric random variable with p = 0.2019.

a) P(Y = 5) = $(1 - 0.2019)^4 0.2019 = 0.0819$.

b) E(Y) = 1/0.2019 = 4.95.

4-95. $E(X) = \int_0^{\infty} x\lambda e^{-\lambda x}dx.$ Use integration by parts with u = x and dv = $\lambda e^{-\lambda x}$.

Then, $E(X) = -xe^{-\lambda x}\Big|_0^{\infty} + \int_0^{\infty} e^{-\lambda x}dx = \dfrac{-e^{-\lambda x}}{\lambda}\Big|_0^{\infty} = 1/\lambda$

V(X) = $\int_0^{\infty} (x - \frac{1}{\lambda})^2 \lambda e^{-\lambda x}dx.$ Use integration by parts with $u = (x - \frac{1}{\lambda})^2$ and $dv = \lambda e^{-\lambda x}$. Then,

$$V(X) = -(x - \tfrac{1}{\lambda})^2 e^{-\lambda x}\Big|_0^{\infty} + 2\int_0^{\infty} (x - \tfrac{1}{\lambda})e^{-\lambda x}dx = (\tfrac{1}{\lambda})^2 + \tfrac{2}{\lambda}\int_0^{\infty} (x - \tfrac{1}{\lambda})\lambda e^{-\lambda x}dx$$

The last integral is seen to be zero from the definition of E(X). Therefore, V(X) = $\left(\frac{1}{\lambda}\right)^2$.

Section 4-10

4-96 a) The time until the tenth call is an Erlang random variable with $\lambda = 5$ calls per minute and r = 10.

b) E(X) = 10/5 = 2 minutes. V(X) = 10/25 = 0.4 minutes.

c) Because a Poisson process is memoryless, the mean time is 1/5=0.2 minutes or 12 seconds

4-97. Let Y denote the number of calls in one minute. Then, Y is a Poisson random variable with $\lambda = 5$ calls per minute.

a) P(Y = 4) = $\dfrac{e^{-5}5^4}{4!} = 0.1755$

b) P(Y > 2) = 1 - $P(Y \leq 2) = 1 - \dfrac{e^{-5}5^0}{0!} - \dfrac{e^{-5}5^1}{1!} - \dfrac{e^{-5}5^2}{2!} = 0.8754$.

Let W denote the number of one minute intervals out of 10 that contain more than 2 calls. Because the calls are a Poisson process, W is a binomial random variable with n = 10 and p = 0.8754.

Therefore, P(W = 10) = $\binom{10}{10} 0.8754^{10}(1 - 0.8754)^0 = 0.2643$.

4-98. Let X denote the pounds of material to obtain 15 particles. Then, X has an Erlang distribution with r = 15 and $\lambda = 0.01$.

a) E(X) = $\dfrac{r}{\lambda} = \dfrac{15}{0.01} = 1500$ pounds.

b) V(X) = $\dfrac{15}{0.01^2} = 150{,}000$ and $\sigma_X = \sqrt{150{,}000} = 387.3$ pounds.

4-99 Let X denote the time between failures of a laser. X is exponential with a mean of 25,000.

a.) Expected time until the second failure $E(X) = r/\lambda = 2/0.00004 = 50{,}000$ hours

b.) N=no of failures in 50000 hours

$$E(N) = \frac{50000}{25000} = 2$$

$$P(N \leq 2) = \sum_{k=0}^{2} \frac{e^{-2}(2)^k}{k!} = 0.6767$$

4-100 Let X denote the time until 5 messages arrive at a node. Then, X has an Erlang distribution with r = 5 and $\lambda = 30$ messages per minute.

a) E(X) = 5/30 = 1/6 minute = 10 seconds.

b)V(X) = $\dfrac{5}{30^2} = 1/180$ minute^2 = 1/3 second and $\sigma_X = 0.0745$ minute = 4.472 seconds.

c) Let Y denote the number of messages that arrive in 10 seconds. Then, Y is a Poisson random variable with $\lambda = 30$ messages per minute = 5 messages per 10 seconds.

$$P(Y \geq 5) = 1 - P(Y \leq 4) = 1 - \left[\frac{e^{-5}5^0}{0!} + \frac{e^{-5}5^1}{1!} + \frac{e^{-5}5^2}{2!} + \frac{e^{-5}5^3}{3!} + \frac{e^{-5}5^4}{4!}\right]$$

$$= 0.5595$$

d) Let Y denote the number of messages that arrive in 5 seconds. Then, Y is a Poisson random variable with $\lambda = 2.5$ messages per 5 seconds.

$$P(Y \geq 5) = 1 - P(Y \leq 4) = 1 - 0.8912 = 0.1088$$

4-101. Let X denote the number of bits until five errors occur. Then, X has an Erlang distribution with r = 5 and $\lambda = 10^{-5}$ error per bit.

a) E(X) = $\dfrac{r}{\lambda} = 5\times 10^5$ bits.

b) V(X) = $\dfrac{r}{\lambda^2} = 5\times 10^{10}$ and $\sigma_X = \sqrt{5\times 10^{10}} = 223607$ bits.

c) Let Y denote the number of errors in 10^5 bits. Then, Y is a Poisson random variable with

$\lambda = 1/10^5 = 10^{-5}$ error per bit = 1 error per 10^5 bits.

$$P(Y \ge 3) = 1 - P(Y \le 2) = 1 - \left[\tfrac{e^{-1}1^0}{0!} + \tfrac{e^{-1}1^1}{1!} + \tfrac{e^{-1}1^2}{2!}\right] = 0.0803$$

4-102 $\lambda = 1/20 = 0.05 \quad r = 100$

a) $E(X) = r/\lambda = 100/.05 = 5$ minutes

b) 4 min - 2.5 min=1.5 min

c)Let Y be the number of calls before 15 seconds $\lambda = 0.25 * 20 = 5$

$$P(Y > 3) = 1 - P(X \le 2) = 1 - \left[\tfrac{e^{-5}5^0}{0!} + \tfrac{e^{-5}5^1}{1!} + \tfrac{e^{-5}5^2}{2!}\right] = 1 - .1247 = 0.8753$$

4-103. a) Let X denote the number of customers that arrive in 10 minutes. Then, X is a Poisson random variable with $\lambda = 0.2$ arrivals per minute = 2 arrivals per 10 minutes.

$$P(X > 3) = 1 - P(X \le 3) = 1 - \left[\tfrac{e^{-2}2^0}{0!} + \tfrac{e^{-2}2^1}{1!} + \tfrac{e^{-2}2^2}{2!} + \tfrac{e^{-2}2^3}{3!}\right] = 0.1429$$

b) Let Y denote the number of customers that arrive in 15 minutes. Then, Y is a Poisson random variable with $\lambda = 3$ arrivals per 15 minutes.

$$P(X \ge 5) = 1 - P(X \le 4) = 1 - \left[\tfrac{e^{-3}3^0}{0!} + \tfrac{e^{-3}3^1}{1!} + \tfrac{e^{-3}3^2}{2!} + \tfrac{e^{-3}3^3}{3!} + \tfrac{e^{-3}3^4}{4!}\right] = 0.1847$$

4-104. Let X denote the time in days until the fourth problem. Then, X has an Erlang distribution with r = 4 and $\lambda = 1/30$ problem per day.

a) E(X) = $\dfrac{4}{30^{-1}} = 120$ days.

b) Let Y denote the number of problems in 120 days. Then, Y is a Poisson random variable with $\lambda = 4$ problems per 120 days.

$$P(Y < 4) = \left[\tfrac{e^{-4}4^0}{0!} + \tfrac{e^{-4}4^1}{1!} + \tfrac{e^{-4}4^2}{2!} + \tfrac{e^{-4}4^3}{3!}\right] = 0.4335$$

4-105. a) $\Gamma(6) = 5! = 120$

b) $\Gamma(\frac{5}{2}) = \frac{3}{2}\Gamma(\frac{3}{2}) = \frac{3}{2}\frac{1}{2}\Gamma(\frac{1}{2}) = \frac{3}{4}\pi^{1/2} = 1.32934$

c) $\Gamma(\frac{9}{2}) = \frac{7}{2}\Gamma(\frac{7}{2}) = \frac{7}{2}\frac{5}{2}\frac{3}{2}\frac{1}{2}\Gamma(\frac{1}{2}) = \frac{105}{16}\pi^{1/2} = 11.6317$

4-106 $\Gamma(r) = \int_0^\infty x^{r-1}e^{-x}dx$. Use integration by parts with $u = x^{r-1}$ and dv =e^{-x}. Then,

$$\Gamma(r) = -x^{r-1}e^{-x}\Big|_0^\infty + (r-1)\int_0^\infty x^{r-2}e^{-x}dx = (r-1)\Gamma(r-1).$$

4-107 $\int_0^\infty f(x;\lambda,r)dx = \int_0^\infty \frac{\lambda^r x^{r-1}e^{-\lambda x}}{\Gamma(r)}dx$. Let y = λx, then the integral is $\int_0^\infty \frac{\lambda\, y^{r-1}e^{-y}}{\Gamma(r)}\frac{dy}{\lambda}$. From the definition of $\Gamma(r)$, this integral is recognized to equal 1.

4-108. If X is a chi-square random variable, then X is a special case of a gamma random variable. Now, E(X) = $\frac{r}{\lambda} = \frac{(7/2)}{(1/2)} = 7$ and $V(X) = \frac{r}{\lambda^2} = \frac{(7/2)}{(1/2)^2} = 14$.

Section 4-11

4-109. β=0.2 and δ=100 hours

$$E(X) = 100\Gamma(1+\tfrac{1}{0.2}) = 100\times 5! = 12{,}000$$

$$V(X) = 100^2\Gamma(1+\tfrac{2}{0.2}) - 100^2[\Gamma(1+\tfrac{1}{0.2})]^2 = 3.61\times 10^{10}$$

4-110. a) $P(X < 10000) = F_X(10000) = 1 - e^{-100^{0.2}} = 1 - e^{-2.512} = 0.9189$

b) $P(X > 5000) = 1 - F_X(5000) = e^{-50^{0.2}} = 0.1123$

4-111. Let X denote lifetime of a bearing. β=2 and δ=10000 hours

a) $P(X > 8000) = 1 - F_X(8000) = e^{-\left(\frac{8000}{10000}\right)^2} = e^{-0.8^2} = 0.5273$

b)

$$E(X) = 10000\Gamma(1+\tfrac{1}{2}) = 10000\Gamma(1.5)$$

$$= 10000(0.5)\Gamma(0.5) = 5000\sqrt{\pi} = 8862.3$$

= 8862.3 hours

c) Let Y denote the number of bearings out of 10 that last at least 8000 hours. Then, Y is a

binomial random variable with n = 10 and p = 0.5273.

$$P(Y = 10) = \binom{10}{10}0.5273^{10}(1-0.5273)^0 = 0.00166 .$$

4-112 a.) $E(X) = \delta\Gamma(1+\frac{1}{\beta}) = 900\Gamma(1+1/3) = 900\Gamma(4/3) = 900(0.89298). = 803.68$ hours

b.)

$$V(X) = \delta^2\Gamma(1+\tfrac{2}{\beta}) - \delta^2\left[\Gamma(1+\tfrac{2}{\beta})\right]^2 = 900^2\Gamma(1+\tfrac{2}{3}) - 900^2\left[\Gamma(1+\tfrac{1}{3})\right]^2$$

$$= 900^2(0.90274) - 900^2(0.89298)^2 = 85314.64 \text{ hours}^2$$

c.) $P(X < 500) = F_X(500) = 1 - e^{-\left(\frac{500}{900}\right)^3} = 0.1576$

4-113. Let *X* denote the lifetime.

a) $E(X) = \delta\Gamma(1+\frac{1}{0.5}) = \delta\Gamma(3) = 2\delta = 600$. Then $\delta = 300$. Now,

P(X > 500) = $e^{-(\frac{500}{300})^{0.5}} = 0.2750$

b) P(X < 400) = $1 - e^{-(\frac{400}{300})^{0.5}} = 0.6848$

4-114 Let X denote the lifetime

a) $E(X) = 700\Gamma(1+\frac{1}{2}) = 620.4$

b)

$$V(X) = 700^2\Gamma(2) - 700^2[\Gamma(1.5)]^2$$
$$= 700^2(1) - 700^2(0.25\pi) = 105{,}154.9$$

c) P(X > 620.4) = $e^{-\left(\frac{620.4}{700}\right)^2} = 0.4559$

4-115 a.)β=2, δ=500

$$E(X) = 500\Gamma(1+\frac{1}{2}) = 500\Gamma(1.5)$$
$$= 500(0.5)\Gamma(0.5) = 250\sqrt{\pi} = 443.11$$

= 443.11 hours

b.) $V(X) = 500^2\Gamma(1+\ 1) - 500^2[\Gamma(1+\ \frac{1}{2})]^2$

$$= 500^2\Gamma(2) - 500^2[\Gamma(1.5)]^2 = 53650.5$$

c.) P(X < 250) = F(250)=$1 - e^{-(\frac{250}{500})^{25}} = 1 - 0.7788 = 0.2212$

4-116 If X is a Weibull random variable with β=1 and δ=1000, the distribution of X is the exponential distribution with λ=.001.

$$f(x) = \left(\frac{1}{1000}\right)\left(\frac{x}{1000}\right)^0 e^{-\left(\frac{x}{1000}\right)^1} \text{ for } x > 0$$
$$= 0.001e^{-0.001x} \text{ for } x > 0$$

The mean of X is E(X) = 1/λ = 1000.

Section 4-11

4-117 X is a lognormal distribution with θ=5 and ω^2=9

a.)

$$P(X < 13300) = P(e^W < 13300) = P(W < \ln(13300)) = \Phi\left(\frac{\ln(13330) - 5}{3}\right)$$

$$= \Phi(1.50) = 0.9332$$

b.) Find the value for which P(X≤x)=0.95

$$P(X \le x) = P(e^W \le x) = P(W < \ln(x)) = \Phi\left(\frac{\ln(x) - 5}{3}\right) = 0.95$$

$$\frac{\ln(x) - 5}{3} = 1.65 \quad x = e^{1.65(3)+5} = 20952.2$$

c.) $\mu = E(X) = e^{\theta + \omega^2/2} = e^{5+9/2} = e^{9.5} = 13359.7$

$$V(X) = e^{2\theta + \omega^2}(e^{\omega^2} - 1) = e^{10+9}(e^9 - 1) = e^{19}(e^9 - 1) = 1.45x10^{12}$$

4-118 a.) X is a lognormal distribution with θ=-2 and ω^2=9

$$P(500 < X < 1000) = P(500 < e^W < 1000) = P(\ln(500) < W < \ln(1000))$$

$$= \Phi\left(\frac{\ln(1000) + 2}{3}\right) - \Phi\left(\frac{\ln(500) + 2}{3}\right) = \Phi(2.97) - \Phi(2.74) = 0.0016$$

b.) $P(X < x) = P(e^W \le x) = P(W < \ln(x)) = \Phi\left(\frac{\ln(x) + 2}{3}\right) = 0.1$

$$\frac{\ln(x) + 2}{3} = -1.28 \qquad x = e^{-1.28(3)-2} = 0.0029$$

c.) $\mu = E(X) = e^{\theta + \omega^2/2} = e^{-2+9/2} = e^{2.5} = 12.1825$

$$V(X) = e^{2\theta + \omega^2}(e^{\omega^2} - 1) = e^{-4+9}(e^9 - 1) = e^5(e^9 - 1) = 1{,}202{,}455.87$$

4-119 a.) X is a lognormal distribution with θ=2 and ω^2=4

$$P(X < 500) = P(e^W < 500) = P(W < \ln(500)) = \Phi\left(\frac{\ln(500) - 2}{2}\right)$$

$$= \Phi(2.11) = 0.9826$$

b.)

$$P(X < 15000 \mid X > 1000) = \frac{P(1000 < X < 1500)}{P(X > 1000)}$$

$$= \frac{\left[\Phi\left(\frac{\ln(1500) - 2}{2}\right) - \Phi\left(\frac{\ln(1000) - 2}{2}\right)\right]}{\left[1 - \Phi\left(\frac{\ln(1000) - 2}{2}\right)\right]}$$

$$= \frac{\Phi(2.66) - \Phi(2.45)}{(1 - \Phi(2.45))} = \frac{0.9961 - 0.9929}{(1 - 0.9929)} = 0.0032 / 0.007 = 0.45$$

c.) The product has degraded over the first 1000 hours, so the probability of it lasting another 500 hours is very low.

4-120 X is a lognormal distribution with θ=0.5 and ω^2=1

a)

$$P(X > 10) = P(e^W > 10) = P(W > \ln(10)) = 1 - \Phi\left(\frac{\ln(10) - 0.5}{1}\right)$$

$$= 1 - \Phi(1.80) = 1 - 0.96407 = 0.03593$$

b.) $$P(X \le x) = P(e^W \le x) = P(W < \ln(x)) = \Phi\left(\frac{\ln(x) - 0.5}{1}\right) = 0.50$$

$$\frac{\ln(x) - 0.5}{1} = 0 \quad x = e^{0(1)+0.5} = 1.65 \text{ seconds}$$

c.) $$\mu = E(X) = e^{\theta+\omega^2/2} = e^{0.5+1/2} = e^1 = 2.7183$$

$$V(X) = e^{2\theta+\omega^2}(e^{\omega^2} - 1) = e^{1+1}(e^1 - 1) = e^2(e^1 - 1) = 12.6965$$

4-121 Find the values of θ and ω^2 given that E(X) = 100 and V(X) = 85,000

$$100 = e^{\theta+\omega^2/2} \qquad 85000 = e^{2\theta+\omega^2}(e^{\omega^2} - 1)$$

let $x = e^{\theta}$ and $y = e^{\omega^2}$ then (1) $100 = x\sqrt{y}$ and (2) $85000 = x^2y(y-1) = x^2y^2 - x^2y$

Square (1) $10000 = x^2 y$ and substitute into (2)

$$85000 = 10000(y-1)$$

$$y = 9.5$$

Substitute y into (1) and solve for x $x = \dfrac{100}{\sqrt{9.5}} = 32.444$

$\theta = \ln(32.444) = 3.48$ and $\omega^2 = \ln(9.5) = 2.25$

4-122 a.) Find the values of θ and ω^2 given that E(X) = 10000 and σ= 20,000

$$10000 = e^{\theta+\omega^2/2} \qquad 20000^2 = e^{2\theta+\omega^2}(e^{\omega^2} - 1)$$

let $x = e^{\theta}$ and $y = e^{\omega^2}$ then (1) $10000 = x\sqrt{y}$ and

(2) $20000^2 = x^2y(y-1) = x^2y^2 - x^2y$

Square (1) $10000^2 = x^2 y$ and substitute into (2)

$$20000^2 = 10000^2(y-1)$$

$$y = 5$$

Substitute y into (1) and solve for x $x = \dfrac{10000}{\sqrt{5}} = 4472.1360$

$\theta = \ln(4472.1360) = 8.4056$ and $\omega^2 = \ln(5) = 1.6094$

b.)

$$P(X > 10000) = P(e^W > 10000) = P(W > \ln(10000)) = 1 - \Phi\left(\frac{\ln(10000) - 8.4056}{1.2686}\right)$$

$$= 1 - \Phi(0.63) = 1 - 0.7357 = 0.2643$$

c.) $P(X > x) = P(e^W > x) = P(W > \ln(x)) = \Phi\left(\dfrac{\ln(x) - 8.4056}{1.2686}\right) = 0.1$

$$\frac{\ln(x) - 8.4056}{1.2686} = -1.28 \quad x = e^{-1.280(1.2686)+8.4056} = 881.65 \text{ hours}$$

4-123 Let X ~N(μ, σ^2), then Y = e^X follows a lognormal distribution with mean μ and variance σ^2. By definition, $F_Y(y) = P(Y \le y) = P(e^X < y) = P(X < \log y) = F_X(\log y) = \Phi\left(\frac{\log y - \mu}{\sigma}\right)$.

Since Y = e^X and X ~ N(μ, σ^2), we can show that $f_Y(Y) = \frac{1}{y} f_X(\log y)$

Finally, $f_Y(y) = \dfrac{\partial F_Y(y)}{\partial y} = \dfrac{\partial F_X(\log y)}{\partial y} = \dfrac{1}{y} f_X(\log y) = \dfrac{1}{y} \cdot \dfrac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{\log y - \mu}{2\sigma}\right)^2}$.

Supplemental Exercises

4-124 a) $P(X < 2.5) = \int_2^{2.5} (0.5x - 1)dx = \left(0.5\frac{x^2}{2} - x\right)\Big|_2^{2.5} = 0.0625$

b) $P(X > 3) = \int_3^4 (0.5x - 1)dx = 0.5\frac{x^2}{2} - x\Big|_3^4 = 0.75$

c) $P(2.5 < X < 3.5) = \int_{2.5}^{3.5} (0.5x - 1)dx = 0.5\frac{x^2}{2} - x\Big|_{2.5}^{3.5} = 0.5$

4-125 $F(x) = \int_2^x (0.5t - 1)dt = 0.5\frac{t^2}{2} - t\Big|_2^x = \frac{x^2}{4} - x + 1$. Then,

$$F(x) = \begin{cases} 0, & x < 2 \\ \frac{x^2}{4} - x + 1, & 2 \le x < 4 \\ 1, & 4 \le x \end{cases}$$

4-126 $E(X) = \int_2^4 x(0.5x - 1)dx = 0.5\frac{x^3}{3} - \frac{x^2}{2}\Big|_2^4 = \frac{32}{3} - 8 - (\frac{4}{3} - 2) = \frac{10}{3}$

$$V(X) = \int_2^4 (x - \tfrac{10}{3})^2 (0.5x - 1)dx = \int_2^4 (x^2 - \tfrac{20}{3}x + \tfrac{100}{9})(0.5x - 1)dx$$

$$= \int_2^4 (0.5x^3 - \tfrac{13}{3}x^2 + \tfrac{110}{9}x - \tfrac{100}{9})dx = \tfrac{x^4}{8} - \tfrac{13}{9}x^3 + \tfrac{55}{9}x^2 - \tfrac{100}{9}x\Big|_2^4$$

$$= 0.2222$$

4-127. Let X denote the time between calls. Then, $\lambda = 1/E(X) = 0.1$ calls per minute.

a) $P(X < 5) = \int_0^5 0.1e^{-0.1x}dx = -e^{-0.1x}\Big|_0^5 = 1 - e^{-0.5} = 0.3935$

b) $P(5 < X < 15) = -e^{-0.1x}\Big|_5^{15} = e^{-0.5} - e^{-1.5} = 0.3834$

c) P(X < x) = 0.9. Then, $P(X < x) = \int_0^x 0.1e^{-0.1t}dt = 1 - e^{-0.1x} = 0.9$. Now, x = 23.03 minutes.

4-128 a) This answer is the same as part a. of Exercise 4-127.

$$P(X < 5) = \int_0^5 0.1e^{-0.1x}dx = -e^{-0.1x}\Big|_0^5 = 1 - e^{-0.5} = 0.3935$$

b) This is the probability that there are no calls over a period of 5 minutes. Because a Poisson process is memoryless, it does not matter whether or not the intervals are consecutive.

$$P(X > 5) = \int_5^\infty 0.1e^{-0.1x}dx = -e^{-0.1x}\Big|_5^\infty = e^{-0.5} = 0.6065$$

4-129. a) Let Y denote the number of calls in 30 minutes. Then, Y is a Poisson random variable with $\lambda = 3$. $P(Y \le 2) = \frac{e^{-3}3^0}{0!} + \frac{e^{-3}3^1}{1!} + \frac{e^{-3}3^2}{2!} = 0.423$.

b) Let W denote the time until the fifth call. Then, W has an Erlang distribution with $\lambda = 0.1$ and r = 5.
E(W) = 5/0.1 = 50 minutes.

4-130 Let X denote the lifetime. Then $\lambda = 1/E(X) = 1/6$.

$$P(X < 3) = \int_0^3 \tfrac{1}{6}e^{-x/6}dx = -e^{-x/6}\Big|_0^3 = 1 - e^{-0.5} = 0.3935.$$

4-131. Let W denote the number of CPUs that fail within the next three years. Then, W is a binomial random variable with n = 10 and p = 0.3935 (from Exercise 4-130). Then,

$$P(W \ge 1) = 1 - P(W = 0) = 1 - \binom{10}{0}0.3935^0(1 - 0.3935)^{10} = 0.9933.$$

4-132 X is a lognormal distribution with θ=0 and ω^2=4

a.)

$$P(10 < X < 50) = P(10 < e^W < 50) = P(\ln(10) < W > \ln(50))$$

$$= \Phi\left(\frac{\ln(50)-0}{2}\right) - \Phi\left(\frac{\ln(10)-0}{2}\right)$$

$$= \Phi(1.96) - \Phi(1.15) = 0.975002 - 0.874928 = 0.10007$$

b.) $P(X < x) = P(e^W < x) = P(W < \ln(x)) = \Phi\left(\frac{\ln(x)-0}{2}\right) = 0.05$

$$\frac{\ln(x)-0}{2} = -1.64 \quad x = e^{-1.64(2)} = 0.0376$$

c.) $\mu = E(X) = e^{\theta+\omega^2/2} = e^{0+4/2} = e^2 = 7.389$

$$V(X) = e^{2\theta+\omega^2}(e^{\omega^2}-1) = e^{0+4}(e^4-1) = e^4(e^4-1) = 2926.40$$

4-133 a.) Find the values of θand ω^2 given that E(X) = 50 and V(X) = 4000

$$50 = e^{\theta+\omega^2/2} \qquad 4000 = e^{2\theta+\omega^2}(e^{\omega^2}-1)$$

let $x = e^{\theta}$ and $y = e^{\omega^2}$ then (1) $50 = x\sqrt{y}$ and (2) $4000 = x^2y(y-1) = x^2y^2 - x^2y$

Square (1) for x $x = \frac{50}{\sqrt{y}}$ and substitute into (2)

$$4000 = \left(\frac{50}{\sqrt{y}}\right)^2 y^2 - \left(\frac{50}{\sqrt{y}}\right)^2 y = 2500(y-1)$$

$$y = 2.6$$

substitute y back in to (1) and solve for x $x = \frac{50}{\sqrt{2.6}} = 31$

$\theta = \ln(31) = 3.43$ and $\omega^2 = \ln(2.6) = 0.96$

b.)

$$P(X < 150) = P(e^W < 150) = P(W < \ln(150)) = \Phi\left(\frac{\ln(150)-3.43}{0.98}\right)$$

$$= \Phi(1.61) = 0.946301$$

4-134 Let X denote the number of fibers visible in a grid cell. Then, X has a Poisson distribution and $\lambda = 100$ fibers per cm^2 = 80,000 fibers per sample = 0.5 fibers per grid cell.

a) $P(X \geq 1) = 1 - P(X) = 1 - \frac{e^{-0.5} 0.5^0}{0!} = 0.3935$.

b) Let W denote the number of grid cells examined until 10 contain fibers. If the number of fibers have a Poisson distribution, then the number of fibers in each grid cell are independent. Therefore, W has a negative binomial distribution with p = 0.3935. Consequently, E(W) = 10/0.3935 = 25.41 cells.

c) V(W) = $\frac{10(1-0.3935)}{0.3935^2}$. Therefore, $\sigma_W = 6.25$ cells.

4-135. Let X denote the height of a plant.

a) P(X>2.25) = $P\left(Z > \frac{2.25-2.5}{0.5}\right)$ = P(Z > -0.5) = 1 - P(Z $\leq$ -0.5) = 0.6915

b) P(2.0 < X < 3.0) = $P\left(\frac{2.0-2.5}{0.5} < Z < \frac{3.0-2.5}{0.5}\right)$ =P(-1 < Z < 1) = 0.683

c.)P(X > x) = 0.90 = $P\left(Z > \frac{x-2.5}{0.5}\right)$ = 0.90 and $\frac{x-2.5}{0.5}$ = -1.28.

Therefore, x = 1.86.

4-136 a) P(X > 3.5) from part a. of Exercise 4-135 is 0.023.

b) Yes, because the probability of a plant growing to a height of 3.5 centimeters or more without irrigation is low.

4-137. Let X denote the thickness.

a) P(X > 5.5) = $P\left(Z > \frac{5.5-5}{0.2}\right)$ = P(Z > 2.5) = 0. 0062

b) P(4.5 < X < 5.5) = $P\left(\frac{4.5-5}{0.2} < Z < \frac{5.5-5}{0.2}\right)$ = P (-2.5 < Z < 2.5) = 0.9876

Therefore, the proportion that do not meet specifications is
1 – P(4.5 < X < 5.5) = 0.012.

c) If P(X < x) = 0.95, then $P\left(Z > \frac{x-5}{0.2}\right)$ = 0.95. Therefore, $\frac{x-5}{0.2}$ = 1.65 and x = 5.33.

4-138 Let X denote the dot diameter. If P(0.0014 < X < 0.0026) = 0.9973, then

$$P\left(\frac{0.0014-0.002}{\sigma} < Z < \frac{0.0026-0.002}{\sigma}\right) = P\left(\frac{-0.0006}{\sigma} < Z < \frac{0.0006}{\sigma}\right) = 0.9973.$$

Therefore, $\frac{0.0006}{\sigma}$ = 3 and $\sigma = 0.0002$.

4-139. If P(0.002-x < X < 0.002+x), then P(-x/0.0004 < Z < x/0.0004) = 0.9973. Therefore, x/0.0004 = 3 and x = 0.0012. The specifications are from 0.0008 to 0.0032.

4-140 Let X denote the life.

a) $P(X < 5800) = P(Z < \frac{5800-7000}{600}) = P(Z < -2) = 1 - P(Z \le 2) = 0.023$

d) If P(X > x) = 0.9, then P(Z < $\frac{x-7000}{600}$) = -1.28. Consequently, $\frac{x-7000}{600}$ = -1.28 and x = 6232 hours.

4-141. If P(X > 10,000) = 0.99, then P(Z > $\frac{10,000-\mu}{600}$) = 0.99. Therefore, $\frac{10,000-\mu}{600}$ = -2.33 and $\mu = 11,398$.

4-142 The probability a product lasts more than 10000 hours is $[P(X > 10000)]^3$, by independence. If $[P(X > 10000)]^3$ = 0.99, then P(X > 10000) = 0.9967.

Then, P(X > 10000) = $P(Z > \frac{10000-\mu}{600}) = 0.9967$. Therefore, $\frac{10000-\mu}{600}$ = -2.72 and $\mu = 11,632$ hours.

4-143 X is an exponential distribution with E(X) = 7000 hours

a.) $P(X < 5800) = \int_0^{5800} \frac{1}{7000} e^{-\frac{x}{7000}} dx = 1 - e^{-\left(\frac{5800}{7000}\right)} = 0.5633$

b.) $P(X > x) = \int_x^{\infty} \frac{1}{7000} e^{-\frac{x}{7000}} dx = 0.9$ Therefore, $e^{-\frac{x}{7000}} = 0.9$

and $x = -7000\ln(0.9) = 737.5$ hours

4-144 Find the values of θ and ω^2 given that E(X) = 7000 and σ= 600

$$7000 = e^{\theta+\omega^2/2} \qquad 600^2 = e^{2\theta+\omega^2}(e^{\omega^2}-1)$$

let $x = e^{\theta}$ and $y = e^{\omega^2}$ then (1) $7000 = x\sqrt{y}$ and

(2) $600^2 = x^2 y(y-1) = x^2y^2 - x^2y$

Square (1) $7000^2 = x^2 y$ and substitute into (2)

$$600^2 = 7000^2(y-1)$$

$$y = 1.0073$$

Substitute y into (1) and solve for x $x = \dfrac{7000}{\sqrt{1.0073}} = 6974.6$

$\theta = \ln(6974.6) = 8.850$ and $\omega^2 = \ln(1.0073) = 0.0073$

a.)

$$P(X < 5800) = P(e^W < 5800) = P(W < \ln(5800)) = \Phi\left(\frac{\ln(5800) - 8.85}{0.0854}\right)$$

$$= \Phi(-2.16) = 0.015$$

b.) $P(X > x) = P(e^W > x) = P(W > \ln(x)) = \Phi\left(\dfrac{\ln(x) - 8.85}{0.0854}\right) = 0.9$

$\dfrac{\ln(x) - 8.85}{0.0854} = -1.28 \quad x = e^{-1.28(0.0854)+8.85} = 6252.20$ hours

4-145. a) Using the normal approximation to the binomial with n = 50*36*36=64,800, and p = 0.0001 we have: E(X)=64800(0.0001)=6.48

$$P(X \ge 1) \cong P\left(\frac{X - np}{\sqrt{np(1-p)}} \ge \frac{1 - 6.48}{\sqrt{64800(0.0001)(0.9999)}}\right)$$

$$= P(Z > -2.15) = 1 - 0.01578 = 0.98422$$

b)

$$P(X \ge 4) \cong P\left(\frac{X - np}{\sqrt{np(1-p)}} \ge \frac{4 - 6.48}{\sqrt{64800(0.0001)(0.9999)}}\right)$$

$$= P(Z \ge -0.97) = 1 - 0.166023 = 0.8340$$

4-146 Using the normal approximation to the binomial with X being the number of people who will be seated.

X ~Bin(200, 0.9).

a) P(X ≤ 185) $= P\left(\dfrac{X-np}{\sqrt{np(1-p)}} \geq \dfrac{185-180}{\sqrt{200(0.9)(0.1)}}\right) = P(Z \leq 1.18) = 0.8810$

b)

$$P(X < 185)$$
$$= P(X \leq 184) = P\left(\frac{X-np}{\sqrt{np(1-p)}} \geq \frac{184-180}{\sqrt{200(0.9)(0.1)}}\right) = P(Z \leq 0.94) = 0.8264$$

c) P(X ≤ 185) ≅ 0.95, Successively trying various values of n: The number of reservations taken could be reduced to about 198.

n	Z_o	Probability P(Z < Z_0)
190	3.39	0.99965
195	2.27	0.988396
198	**1.61**	**0.946301**

Mind-Expanding Exercises

4-147. a) P(X > x) implies that there are r - 1 or less counts in an interval of length x. Let Y denote the number of counts in an interval of length x. Then, Y is a Poisson random variable with parameter λx. Then,

$$P(X > x) = P(Y \leq r-1) = \sum_{i=0}^{r-1} e^{-\lambda x} \tfrac{(\lambda x)^i}{i!}.$$

b) $P(X \leq x) = 1 - \sum_{i=0}^{r-1} e^{-\lambda x} \tfrac{(\lambda x)^i}{i!}$

c) $f_X(x) = \tfrac{d}{dx} F_X(x) = \lambda e^{-\lambda x} \sum_{i=0}^{r-1} \dfrac{(\lambda x)^i}{i!} - e^{-\lambda x} \sum_{i=0}^{r-1} \lambda i \dfrac{(\lambda x)^i}{i!} = \lambda e^{-\lambda x} \dfrac{(\lambda x)^{r-1}}{(r-1)!}$

4-148. Let X denote the diameter of the maximum diameter bearing. Then, P(X > 1.6) = 1 - $P(X \leq 1.6)$. Also, $X \leq 1.6$ if and only if all the diameters are less than 1.6. Let Y denote the diameter of a bearing. Then, by independence

$$P(X \leq 1.6) = [P(Y \leq 1.6)]^{10} = \left[P(Z \leq \tfrac{1.6-1.5}{0.025})\right]^{10} = 0.999967^{10} = 0.99967$$

Then, P(X > 1.575) = 0.0033.

4-149. a) Quality loss = $Ek(X-m)^2 = kE(X-m)^2 = k\sigma^2$, by the definition of the variance.

b)

$$
\begin{aligned}
Quality\ \ loss &= Ek(X-m)^2 = kE(X-\mu+\mu-m)^2 \\
&= kE[(X-\mu)^2 + (\mu-m)^2 + 2(\mu-m)(X-\mu)] \\
&= kE(X-\mu)^2 + k(\mu-m)^2 + 2k(\mu-m)E(X-\mu).
\end{aligned}
$$

The last term equals zero by the definition of the mean.

Therefore, quality loss = $k\sigma^2 + k(\mu-m)^2$.

4-150. Let X denote the event that an amplifier fails before 60,000 hours. Let A denote the event that an amplifier mean is 20,000 hours. Then A' is the event that the mean of an amplifier is 50,000 hours. Now, P(E) = P(E|A)P(A) + P(E|A')P(A') and

$$P(E \mid A) = \int_0^{60,000} \tfrac{1}{20,000} e^{-x/20,000} dx = -e^{-x/20,000}\Big|_0^{60,000} = 1 - e^{-3} = 0.9502$$

$$P(E \mid A') = -e^{-x/50,000}\Big|_0^{60,000} = 1 - e^{-6/5} = 0.6988.$$

Therefore, P(E) = 0.9502(0.10) + 0.6988(0.90) = 0.7239

4-151. $P(X < t_1 + t_2 \mid X > t_1) = \frac{P(t_1 < X < t_1 + t_2)}{P(X > t_1)}$ from the definition of conditional probability. Now,

$$P(t_1 < X < t_1 + t_2) = \int_{t_1}^{t_1+t_2} \lambda e^{-\lambda x} dx = -e^{-\lambda x}\Big|_{t_1}^{t_1+t_2} = e^{-\lambda t_1} - e^{-\lambda(t_1+t_2)}$$

$$P(X > t_1) = -e^{-\lambda x}\Big|_{t_1}^{\infty} = e^{-\lambda t_1}$$

Therefore, $P(X < t_1 + t_2 \mid X > t_1) = \dfrac{e^{-\lambda t_1}(1 - e^{-\lambda t_2})}{e^{-\lambda t_1}} = 1 - e^{-\lambda t_2} = P(X < t_2)$

4-152. a)

$$1-P(\mu_0-6\sigma<X<\mu_0+6\sigma)=1-P(-6<Z<6)$$
$$=1.97\times10^{-9}=0.00197\,ppm$$

b)

$$1-P(\mu_0-6\sigma<X<\mu_0+6\sigma)=1-P(-7.5<\tfrac{X-(\mu_0+1.5\sigma)}{\sigma}<4.5)$$
$$=3.4\times10^{-6}=3.4\,ppm$$

c)

$$1-P(\mu_0-3\sigma<X<\mu_0+3\sigma)=1-P(-3<Z<3)$$
$$=.0027=2{,}700\,ppm$$

d)

$$1-P(\mu_0-3\sigma<X<\mu_0+3\sigma)=1-P(-4.5<\tfrac{X-(\mu_0+1.5\sigma)}{\sigma}<1.5)$$
$$=0.0668106=66{,}810.6\,ppm$$

e) If the process is centered six standard deviations away from the specification limits and the process shift, there will be significantly less product loss. If the process is centered only three standard deviations away from the specifications and the process shifts, there could be a great loss of product.

Section 4-8 on CD

S4-1. E(X) = 50(0.1) = 5 and V(X) = 50(0.1)(0.9) = 4.5

a) $P(X\le2)=P(X\le2.5)\cong P(Z\le\frac{2.5-5}{\sqrt{4.5}})=P(Z\le-1.18)=0.119$

b) $P(X\le2)\cong P(Z\le\frac{2-2.5}{\sqrt{4.5}})=P(Z\le-0.24)=0.206$

c) $P(X\le2)=\binom{50}{0}0.1^0 0.9^{50}+\binom{50}{1}0.1^1 0.9^{49}+\binom{50}{2}0.1^2 0.9^{48}=0.118$

The probability computed using the continuity correction is closer.

d) $P(X<10)=P(X\le9.5)\cong P\left(Z\le\dfrac{9.5-5}{\sqrt{4.5}}\right)=P(Z\le2.12)=0.983$

S4-2. E(X) = 50(0.1) = 5 and V(X) = 50(0.1)(0.9) = 4.5

a) $P(X\ge2)=P(X\ge1.5)\cong P(Z\le\frac{1.5-5}{\sqrt{4.5}})=P(Z\ge-1.65)=0.951$

b) $P(X\ge2)\cong P(Z\ge\frac{2-5}{\sqrt{4.5}})=P(Z\ge-1.414)=0.921$

c) $P(X\ge2)=1-P(X<2)=1-\binom{50}{0}0.1^0 0.9^{50}-\binom{50}{1}0.1^1 0.9^{49}=0.966$

The probability computed using the continuity correction is closer.

d) $P(X>6)=P(X\ge7)=P(X\ge6.5)\cong P(Z\ge0.707)=0.24$

S4-3. E(X) = 50(0.1) = 5 and V(X) = 50(0.1)(0.9) = 4.5

a)

$$P(2 \le X \le 5) = P(1.5 \le X \le 5.5) \cong P(\tfrac{1.5-5}{\sqrt{4.5}} \le Z \le \tfrac{5.5-5}{\sqrt{4.5}})$$

$$= P(-1.65 \le Z \le 0.236) = P(Z \le 0.24) - P(Z \le -1.65)$$

$$= 0.5948 - (1 - 0.95053) = 0.5453$$

b)

$$P(2 \le X \le 5) \cong P(\tfrac{2-5}{\sqrt{4.5}} \le Z \le \tfrac{5-5}{\sqrt{4.5}}) = P(-1.414 \le Z \le 0)$$

$$= 0.5 - P(Z \le -1.414)$$

$$= 0.5 - (1 - 0.921) = 0.421$$

The exact probability is 0.582

S4-4. E(X) = 50(0.1) = 5 and V(X) = 50(0.1)(0.9) = 4.5

a)

$$P(X = 10) = P(9.5 \le X \le 10.5) \cong P(\tfrac{9.5-5}{\sqrt{4.5}} \le Z \le \tfrac{10.5-5}{\sqrt{4.5}})$$

$$= P(2.121 \le Z \le 2.593) = 0.012$$

b)

$$P(X = 5) = P(4.5 \le X \le 5.5) \cong P(\tfrac{4.5-5}{\sqrt{4.5}} \le Z \le \tfrac{5.5-5}{\sqrt{4.5}})$$

$$= P(-0.24 \le Z \le 0.24) = 0.1897$$

S4-5 Let X be the number of chips in the lot that are defective. Then E(X)=1000(0.02)=20 and V(X)=1000(0.02)(0.98)=19.6

a) P(20 ≤ X ≤ 30)=P(19.5 ≤ X ≤ 30.5) =

$$P\left(\frac{19.5-20}{\sqrt{19.6}} \le Z \le \frac{30.5-20}{\sqrt{19.6}}\right) = P(-.11 \le Z \le 2.37) = 0.9911 - 0.4562 = 0.5349$$

b) P(X=20)=P(19.5 ≤ X ≤ 20.5) =P(-0.11 ≤ Z ≤ 0.11)=0.5438-0.4562=0.0876.

c) The answer should be close to the mean. Substituting values close to the mean, we find x=20 gives the maximum probability.