

CHAPTER 8

Section 8-2

- 8-1 a.) The confidence level for $\bar{x} - 2.14\sigma/\sqrt{n} \leq \mu \leq \bar{x} + 2.14\sigma/\sqrt{n}$ is determined by the value of z_0 which is 2.14. From Table II, we find $\Phi(2.14) = P(Z < 2.14) = 0.9838$ and the confidence level is $100(1 - 0.032354) = 96.76\%$.
- b.) The confidence level for $\bar{x} - 2.49\sigma/\sqrt{n} \leq \mu \leq \bar{x} + 2.49\sigma/\sqrt{n}$ is determined by the value of z_0 which is 2.49. From Table II, we find $\Phi(2.49) = P(Z < 2.49) = 0.9936$ and the confidence level is $100(1 - 0.012774) = 98.72\%$.
- c.) The confidence level for $\bar{x} - 1.85\sigma/\sqrt{n} \leq \mu \leq \bar{x} + 1.85\sigma/\sqrt{n}$ is determined by the value of z_0 which is 2.14. From Table II, we find $\Phi(1.85) = P(Z < 1.85) = 0.9678$ and the confidence level is 93.56%.

- 8-2 a.) A $z_\alpha = 2.33$ would give result in a 98% two-sided confidence interval.
 b.) A $z_\alpha = 1.29$ would give result in a 80% two-sided confidence interval.
 c.) A $z_\alpha = 1.15$ would give result in a 75% two-sided confidence interval.

- 8-3 a.) A $z_\alpha = 1.29$ would give result in a 90% one-sided confidence interval.
 b.) A $z_\alpha = 1.65$ would give result in a 95% one-sided confidence interval.
 c.) A $z_\alpha = 2.33$ would give result in a 99% one-sided confidence interval.

- 8-4 a.) 95% CI for μ , $n = 10$, $\sigma = 20$, $\bar{x} = 1000$, $z = 1.96$

$$\bar{x} - z\sigma/\sqrt{n} \leq \mu \leq \bar{x} + z\sigma/\sqrt{n}$$

$$1000 - 1.96(20/\sqrt{10}) \leq \mu \leq 1000 + 1.96(20/\sqrt{10})$$

$$987.6 \leq \mu \leq 1012.4$$

- b.) .95% CI for μ , $n = 25$, $\sigma = 20$, $\bar{x} = 1000$, $z = 1.96$

$$\bar{x} - z\sigma/\sqrt{n} \leq \mu \leq \bar{x} + z\sigma/\sqrt{n}$$

$$1000 - 1.96(20/\sqrt{25}) \leq \mu \leq 1000 + 1.96(20/\sqrt{25})$$

$$992.2 \leq \mu \leq 1007.8$$

- c.) 99% CI for μ , $n = 10$, $\sigma = 20$, $\bar{x} = 1000$, $z = 2.58$

$$\bar{x} - z\sigma/\sqrt{n} \leq \mu \leq \bar{x} + z\sigma/\sqrt{n}$$

$$1000 - 2.58(20/\sqrt{10}) \leq \mu \leq 1000 + 2.58(20/\sqrt{10})$$

$$983.7 \leq \mu \leq 1016.3$$

- d.) 99% CI for μ , $n = 25$, $\sigma = 20$, $\bar{x} = 1000$, $z = 2.58$

$$\bar{x} - z\sigma/\sqrt{n} \leq \mu \leq \bar{x} + z\sigma/\sqrt{n}$$

$$1000 - 2.58(20/\sqrt{25}) \leq \mu \leq 1000 + 2.58(20/\sqrt{25})$$

$$989.7 \leq \mu \leq 1010.3$$

8-5 Find n for the length of the 95% CI to be 40. $Z_{\alpha/2} = 1.96$

$$1/2 \text{ length} = (1.96)(20) / \sqrt{n} = 20$$

$$39.2 = 20\sqrt{n}$$

$$n = \left(\frac{39.2}{20} \right)^2 = 3.84$$

Therefore, $n = 4$.

8-6 Interval (1): $3124.9 \leq \mu \leq 3215.7$ and Interval (2): $3110.5 \leq \mu \leq 3230.1$

Interval (1): half-length = $90.8/2=45.4$ Interval (2): half-length = $119.6/2=59.8$

a.) $\bar{x}_1 = 3124.9 + 45.4 = 3170.3$

$\bar{x}_2 = 3110.5 + 59.8 = 3170.3$ The sample means are the same.

b.) Interval (1): $3124.9 \leq \mu \leq 3215.7$ was calculated with 95% Confidence because it has a smaller half-length, and therefore a smaller confidence interval. The 99% confidence level will make the interval larger.

8-7 a.) The 99% CI on the mean calcium concentration would be longer.

b.) No, that is not the correct interpretation of a confidence interval. The probability that μ is between 0.49 and 0.82 is either 0 or 1.

c.) Yes, this is the correct interpretation of a confidence interval. The upper and lower limits of the confidence limits are random variables.

8-8 95% Two-sided CI on the breaking strength of yarn: where $\bar{x} = 98$, $\sigma = 2$, $n=9$ and $z_{0.025} = 1.96$

$$\bar{x} - z_{0.025} \sigma / \sqrt{n} \leq \mu \leq \bar{x} + z_{0.025} \sigma / \sqrt{n}$$

$$98 - 1.96(2) / \sqrt{9} \leq \mu \leq 98 + 1.96(2) / \sqrt{9}$$

$$96.7 \leq \mu \leq 99.3$$

8-9 95% Two-sided CI on the true mean yield: where $\bar{x} = 90.480$, $\sigma = 3$, $n=5$ and $z_{0.025} = 1.96$

$$\bar{x} - z_{0.025} \sigma / \sqrt{n} \leq \mu \leq \bar{x} + z_{0.025} \sigma / \sqrt{n}$$

$$90.480 - 1.96(3) / \sqrt{5} \leq \mu \leq 90.480 + 1.96(3) / \sqrt{5}$$

$$87.85 \leq \mu \leq 93.11$$

8-10 99% Two-sided CI on the diameter cable harness holes: where $\bar{x} = 1.5045$, $\sigma = 0.01$, $n=10$ and $z_{0.005} = 2.58$

$$\bar{x} - z_{0.005} \sigma / \sqrt{n} \leq \mu \leq \bar{x} + z_{0.005} \sigma / \sqrt{n}$$

$$1.5045 - 2.58(0.01) / \sqrt{10} \leq \mu \leq 1.5045 + 2.58(0.01) / \sqrt{10}$$

$$1.4963 \leq \mu \leq 1.5127$$

- 8-11 a.) 99% Two-sided CI on the true mean piston ring diameter
 For $\alpha = 0.01$, $z_{\alpha/2} = z_{0.005} = 2.58$, and $\bar{x} = 74.036$, $\sigma = 0.001$, $n=15$

$$\bar{x} - z_{0.005} \left(\frac{\sigma}{\sqrt{n}} \right) \leq \mu \leq \bar{x} + z_{0.005} \left(\frac{\sigma}{\sqrt{n}} \right)$$

$$74.036 - 2.58 \left(\frac{0.001}{\sqrt{15}} \right) \leq \mu \leq 74.036 + 2.58 \left(\frac{0.001}{\sqrt{15}} \right)$$

$$74.0353 \leq \mu \leq 74.0367$$

- b.) 95% One-sided CI on the true mean piston ring diameter
 For $\alpha = 0.05$, $z_{\alpha} = z_{0.05} = 1.65$ and $\bar{x} = 74.036$, $\sigma = 0.001$, $n=15$

$$\bar{x} - z_{0.05} \frac{\sigma}{\sqrt{n}} \leq \mu$$

$$74.036 - 1.65 \left(\frac{0.001}{\sqrt{15}} \right) \leq \mu$$

$$74.0356 \leq \mu$$

- 8-12 a.) 95% Two-sided CI on the true mean life of a 75-watt light bulb
 For $\alpha = 0.05$, $z_{\alpha/2} = z_{0.025} = 1.96$, and $\bar{x} = 1014$, $\sigma = 25$, $n=20$

$$\bar{x} - z_{0.025} \left(\frac{\sigma}{\sqrt{n}} \right) \leq \mu \leq \bar{x} + z_{0.025} \left(\frac{\sigma}{\sqrt{n}} \right)$$

$$1014 - 1.96 \left(\frac{25}{\sqrt{20}} \right) \leq \mu \leq 1014 + 1.96 \left(\frac{25}{\sqrt{20}} \right)$$

$$1003 \leq \mu \leq 1025$$

- b.) 95% One-sided CI on the true mean piston ring diameter
 For $\alpha = 0.05$, $z_{\alpha} = z_{0.05} = 1.65$ and $\bar{x} = 1014$, $\sigma = 25$, $n=20$

$$\bar{x} - z_{0.05} \frac{\sigma}{\sqrt{n}} \leq \mu$$

$$1014 - 1.65 \left(\frac{25}{\sqrt{20}} \right) \leq \mu$$

$$1005 \leq \mu$$

- 8-13 a) 95% two sided CI on the mean compressive strength
 $z_{\alpha/2} = z_{0.025} = 1.96$, and $\bar{x} = 3250$, $\sigma^2 = 1000$, $n=12$

$$\bar{x} - z_{0.025} \left(\frac{\sigma}{\sqrt{n}} \right) \leq \mu \leq \bar{x} + z_{0.025} \left(\frac{\sigma}{\sqrt{n}} \right)$$

$$3250 - 1.96 \left(\frac{31.62}{\sqrt{12}} \right) \leq \mu \leq 3250 + 1.96 \left(\frac{31.62}{\sqrt{12}} \right)$$

$$3232.11 \leq \mu \leq 3267.89$$

- b.) 99% Two-sided CI on the true mean compressive strength
 $z_{\alpha/2} = z_{0.005} = 2.58$

$$\bar{x} - z_{0.005} \left(\frac{\sigma}{\sqrt{n}} \right) \leq \mu \leq \bar{x} + z_{0.005} \left(\frac{\sigma}{\sqrt{n}} \right)$$

$$3250 - 2.58 \left(\frac{31.62}{\sqrt{12}} \right) \leq \mu \leq 3250 + 2.58 \left(\frac{31.62}{\sqrt{12}} \right)$$

$$3226.4 \leq \mu \leq 3273.6$$

- 8-14 95% Confident that the error of estimating the true mean life of a 75-watt light bulb is less than 5 hours.
 For $\alpha = 0.05$, $z_{\alpha/2} = z_{0.025} = 1.96$, and $\sigma = 25$, $E=5$

$$n = \left(\frac{z_{\alpha/2} \sigma}{E} \right)^2 = \left(\frac{1.96(25)}{5} \right)^2 = 96.04$$

Always round up to the next number, therefore $n=97$

- 8-15 Set the width to 6 hours with $\sigma = 25$, $z_{0.025} = 1.96$ solve for n.

$$1/2 \text{ width} = (1.96)(25) / \sqrt{n} = 3$$

$$49 = 3\sqrt{n}$$

$$n = \left(\frac{49}{3} \right)^2 = 266.78$$

Therefore, $n=267$.

- 8-16 99% Confident that the error of estimating the true compressive strength is less than 15 psi.
 For $\alpha = 0.01$, $z_{\alpha/2} = z_{0.005} = 2.58$, and $\sigma = 31.62$, $E=15$

$$n = \left(\frac{z_{\alpha/2} \sigma}{E} \right)^2 = \left(\frac{2.58(31.62)}{15} \right)^2 = 29.6 \cong 30$$

Therefore, $n=30$

8-17 To decrease the length of the CI by one half, the sample size must be increased by 4 times (2^2).

$$z_{\alpha/2}\sigma/\sqrt{n} = 0.5l$$

Now, to decrease by half, divide both sides by 2.

$$(z_{\alpha/2}\sigma/\sqrt{n})/2 = (l/2)/2$$

$$(z_{\alpha/2}\sigma/2\sqrt{n}) = l/4$$

$$(z_{\alpha/2}\sigma/\sqrt{2^2n}) = l/4$$

Therefore, the sample size must be increased by 2^2 .

8-18 If n is doubled in Eq 8-7: $\bar{x} - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) \leq \mu \leq \bar{x} + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$

$$\frac{z_{\alpha/2}\sigma}{\sqrt{2n}} = \frac{z_{\alpha/2}\sigma}{1.414\sqrt{n}} = \frac{z_{\alpha/2}\sigma}{1.414\sqrt{n}} = \frac{1}{1.414}\left(\frac{z_{\alpha/2}\sigma}{\sqrt{n}}\right)$$

The interval is reduced by 0.293 29.3%

If n is increased by a factor of 4 Eq 8-7:

$$\frac{z_{\alpha/2}\sigma}{\sqrt{4n}} = \frac{z_{\alpha/2}\sigma}{2\sqrt{n}} = \frac{z_{\alpha/2}\sigma}{2\sqrt{n}} = \frac{1}{2}\left(\frac{z_{\alpha/2}\sigma}{\sqrt{n}}\right)$$

The interval is reduced by 0.5 or $\frac{1}{2}$.

Section 8-3

8-19 $t_{0.025,15} = 2.131$ $t_{0.05,10} = 1.812$ $t_{0.10,20} = 1.325$
 $t_{0.005,25} = 2.787$ $t_{0.001,30} = 3.385$

8-20 a.) $t_{0.025,12} = 2.179$
 b.) $t_{0.025,24} = 2.064$
 c.) $t_{0.005,13} = 3.012$
 d.) $t_{0.0005,15} = 4.073$

8-21 a.) $t_{0.05,14} = 1.761$
 b.) $t_{0.01,19} = 2.539$
 c.) $t_{0.001,24} = 3.467$

8-22 95% confidence interval on mean tire life

$$n = 16 \quad \bar{x} = 60,139.7 \quad s = 3645.94 \quad t_{0.025,15} = 2.131$$

$$\bar{x} - t_{0.025,15} \left(\frac{s}{\sqrt{n}} \right) \leq \mu \leq \bar{x} + t_{0.025,15} \left(\frac{s}{\sqrt{n}} \right)$$

$$60139.7 - 2.131 \left(\frac{3645.94}{\sqrt{16}} \right) \leq \mu \leq 60139.7 + 2.131 \left(\frac{3645.94}{\sqrt{16}} \right)$$

$$58197.33 \leq \mu \leq 62082.07$$

8-23 99% lower confidence bound on mean Izod impact strength

$$n = 20 \quad \bar{x} = 1.25 \quad s = 0.25 \quad t_{0.01,19} = 2.539$$

$$\bar{x} - t_{0.01,19} \left(\frac{s}{\sqrt{n}} \right) \leq \mu$$

$$1.25 - 2.539 \left(\frac{0.25}{\sqrt{20}} \right) \leq \mu$$

$$1.108 \leq \mu$$

8-24 99% confidence interval on mean current required

Assume that the data are normally distributed and that the variance is unknown.

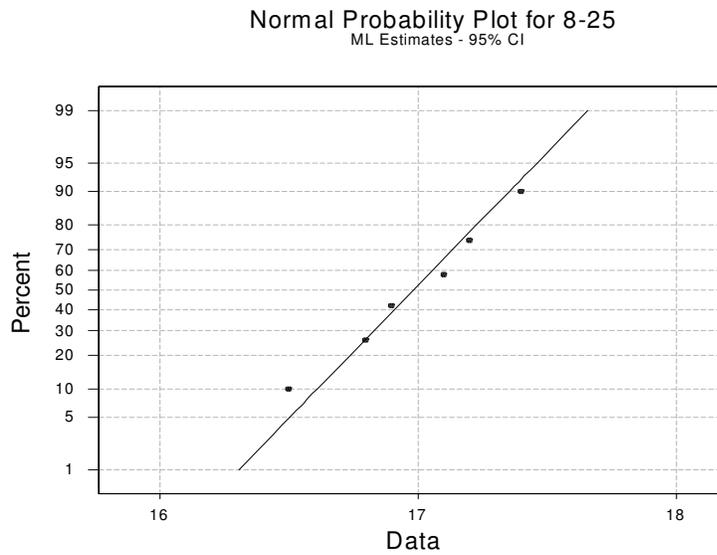
$$n = 10 \quad \bar{x} = 317.2 \quad s = 15.7 \quad t_{0.005,9} = 3.250$$

$$\bar{x} - t_{0.005,9} \left(\frac{s}{\sqrt{n}} \right) \leq \mu \leq \bar{x} + t_{0.005,9} \left(\frac{s}{\sqrt{n}} \right)$$

$$317.2 - 3.250 \left(\frac{15.7}{\sqrt{10}} \right) \leq \mu \leq 317.2 + 3.250 \left(\frac{15.7}{\sqrt{10}} \right)$$

$$301.06 \leq \mu \leq 333.34$$

- 8-25 a.) The data appear to be normally distributed based on examination of the normal probability plot below. Therefore, there is evidence to support that the level of polyunsaturated fatty acid is normally distributed.



- b.) 99% CI on the mean level of polyunsaturated fatty acid.

For $\alpha = 0.01$, $t_{\alpha/2, n-1} = t_{0.005, 5} = 4.032$

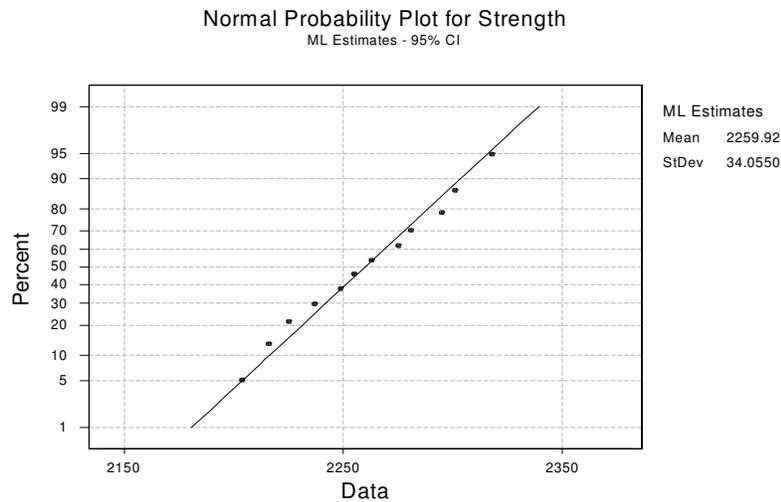
$$\bar{x} - t_{0.005, 5} \left(\frac{s}{\sqrt{n}} \right) \leq \mu \leq \bar{x} + t_{0.005, 5} \left(\frac{s}{\sqrt{n}} \right)$$

$$16.98 - 4.032 \left(\frac{0.319}{\sqrt{6}} \right) \leq \mu \leq 16.98 + 4.032 \left(\frac{0.319}{\sqrt{6}} \right)$$

$$16.455 \leq \mu \leq 17.505$$

The 99% confidence for the mean polyunsaturated fat is (16.455, 17.505). We would look for the true mean to be somewhere in this region.

- 8-26 a.) The data appear to be normally distributed based on examination of the normal probability plot below. Therefore, there is evidence to support that the compressive strength is normally distributed.



- b.) 95% two-sided confidence interval on mean comprehensive strength

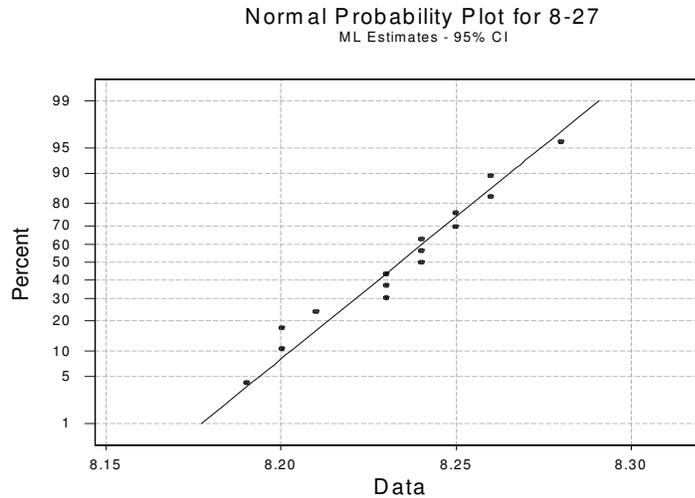
$$n = 12 \quad \bar{x} = 2259.9 \quad s = 35.6 \quad t_{0.025,11} = 2.201$$

$$\begin{aligned} \bar{x} - t_{0.025,11} \left(\frac{s}{\sqrt{n}} \right) &\leq \mu \leq \bar{x} + t_{0.025,11} \left(\frac{s}{\sqrt{n}} \right) \\ 2259.9 - 2.201 \left(\frac{35.6}{\sqrt{12}} \right) &\leq \mu \leq 2259.9 + 2.201 \left(\frac{35.6}{\sqrt{12}} \right) \\ 2237.3 &\leq \mu \leq 2282.5 \end{aligned}$$

- c.) 95% lower-confidence bound on mean strength

$$\begin{aligned} \bar{x} - t_{0.05,11} \left(\frac{s}{\sqrt{n}} \right) &\leq \mu \\ 2259.9 - 1.796 \left(\frac{35.6}{\sqrt{12}} \right) &\leq \mu \\ 2241.4 &\leq \mu \end{aligned}$$

- 8-27 a.) According to the normal probability plot there does not seem to be a severe deviation from normality for this data. This is evident by the fact that the data appears to fall along a straight line.



- b.) 95% two-sided confidence interval on mean rod diameter

For $\alpha = 0.05$ and $n = 15$, $t_{\alpha/2, n-1} = t_{0.025, 14} = 2.145$

$$\bar{x} - t_{0.025, 14} \left(\frac{s}{\sqrt{n}} \right) \leq \mu \leq \bar{x} + t_{0.025, 14} \left(\frac{s}{\sqrt{n}} \right)$$

$$8.23 - 2.145 \left(\frac{0.025}{\sqrt{15}} \right) \leq \mu \leq 8.23 + 2.145 \left(\frac{0.025}{\sqrt{15}} \right)$$

$$8.216 \leq \mu \leq 8.244$$

- 8-28 95% lower confidence bound on mean rod diameter $t_{0.05, 14} = 1.761$

$$\bar{x} - t_{0.05, 14} \left(\frac{s}{\sqrt{n}} \right) \leq \mu$$

$$8.23 - 1.761 \left(\frac{0.025}{\sqrt{15}} \right) \leq \mu$$

$$8.219 \leq \mu$$

The lower bound of the one sided confidence interval is lower than the lower bound of the two-sided confidence interval even though the level of significance is the same. This is because all of the alpha value is in the left tail (or in the lower bound).

- 8-29 95% lower bound confidence for the mean wall thickness
given $\bar{x} = 4.05$ $s = 0.08$ $n = 25$

$$t_{\alpha, n-1} = t_{0.05, 24} = 1.711$$

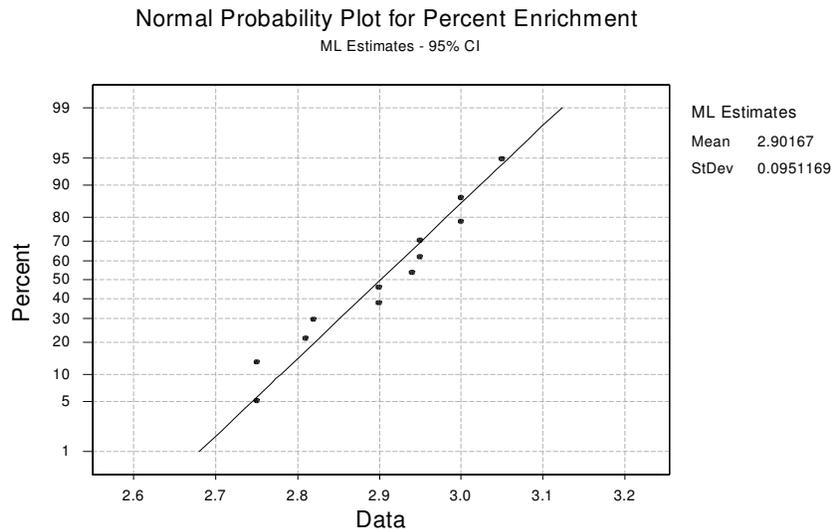
$$\bar{x} - t_{0.05, 24} \left(\frac{s}{\sqrt{n}} \right) \leq \mu$$

$$4.05 - 1.711 \left(\frac{0.08}{\sqrt{25}} \right) \leq \mu$$

$$4.023 \leq \mu$$

It may be assumed that the mean wall thickness will most likely be greater than 4.023 mm.

- 8-30 a.) The data appear to be normally distributed. Therefore, there is no evidence to support that the percentage of enrichment is not normally distributed.



- b.) 99% two-sided confidence interval on mean percentage enrichment

For $\alpha = 0.01$ and $n = 12$, $t_{\alpha/2, n-1} = t_{0.005, 11} = 3.106$, $\bar{x} = 2.9017$ $s = 0.0993$

$$\bar{x} - t_{0.005, 11} \left(\frac{s}{\sqrt{n}} \right) \leq \mu \leq \bar{x} + t_{0.005, 11} \left(\frac{s}{\sqrt{n}} \right)$$

$$2.902 - 3.106 \left(\frac{0.0993}{\sqrt{12}} \right) \leq \mu \leq 2.902 + 3.106 \left(\frac{0.0993}{\sqrt{12}} \right)$$

$$2.813 \leq \mu \leq 2.991$$

8-31 $\bar{x} = 1.10$ $s = 0.015$ $n = 25$
 95% CI on the mean volume of syrup dispensed

For $\alpha = 0.05$ and $n = 25$, $t_{\alpha/2, n-1} = t_{0.025, 24} = 2.064$

$$\bar{x} - t_{0.025, 24} \left(\frac{s}{\sqrt{n}} \right) \leq \mu \leq \bar{x} + t_{0.025, 24} \left(\frac{s}{\sqrt{n}} \right)$$

$$1.10 - 2.064 \left(\frac{0.015}{\sqrt{25}} \right) \leq \mu \leq 1.10 + 2.064 \left(\frac{0.015}{\sqrt{25}} \right)$$

$$1.094 \leq \mu \leq 1.106$$

8-32 90% CI on the mean frequency of a beam subjected to loads

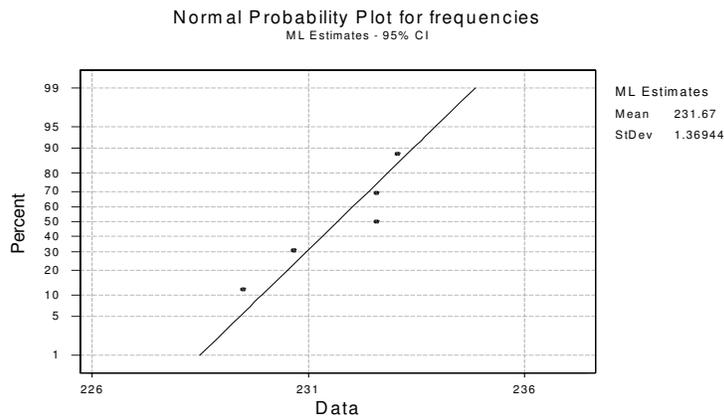
$\bar{x} = 231.67$, $s = 1.53$, $n = 5$, $t_{\alpha/2, n-1} = t_{0.05, 4} = 2.132$

$$\bar{x} - t_{0.05, 4} \left(\frac{s}{\sqrt{n}} \right) \leq \mu \leq \bar{x} + t_{0.05, 4} \left(\frac{s}{\sqrt{n}} \right)$$

$$231.67 - 2.132 \left(\frac{1.53}{\sqrt{5}} \right) \leq \mu \leq 231.67 + 2.132 \left(\frac{1.53}{\sqrt{5}} \right)$$

$$230.2 \leq \mu \leq 233.1$$

By examining the normal probability plot, it appears that the data are normally distributed. There does not appear to be enough evidence to reject the hypothesis that the frequencies are normally distributed.



Section 8-4

$$8-33 \quad \chi_{0.05,10}^2 = 18.31 \quad \chi_{0.025,15}^2 = 27.49 \quad \chi_{0.01,12}^2 = 26.22$$

$$\chi_{0.005,25}^2 = 46.93 \quad \chi_{0.95,20}^2 = 10.85 \quad \chi_{0.99,18}^2 = 7.01 \quad \chi_{0.995,16}^2 = 5.14$$

$$8-34 \quad \text{a.) } \chi_{0.05,24}^2 = 36.42$$

$$\text{b.) } \chi_{0.99,9}^2 = 2.09$$

$$\text{c.) } \chi_{0.95,19}^2 = 10.12 \quad \text{and} \quad \chi_{0.05,19}^2 = 30.14$$

8-35 99% lower confidence bound for σ^2
 For $\alpha = 0.01$ and $n = 15$, $\chi_{\alpha, n-1}^2 = \chi_{0.01,14}^2 = 29.14$

$$\frac{14(0.008)^2}{29.14} \leq \sigma^2$$

$$0.00003075 \leq \sigma^2$$

8-36 95% two sided confidence interval for σ
 $n = 10 \quad s = 4.8$

$$\chi_{\alpha/2, n-1}^2 = \chi_{0.025,9}^2 = 19.02 \quad \text{and} \quad \chi_{1-\alpha/2, n-1}^2 = \chi_{0.975,9}^2 = 2.70$$

$$\frac{9(4.8)^2}{19.02} \leq \sigma^2 \leq \frac{9(4.8)^2}{2.70}$$

$$10.90 \leq \sigma^2 \leq 76.80$$

$$3.30 < \sigma < 8.76$$

8-37 95% lower confidence bound for σ^2 given $n = 16$, $s^2 = (3645.94)^2$
 For $\alpha = 0.05$ and $n = 16$, $\chi_{\alpha, n-1}^2 = \chi_{0.05,15}^2 = 25.00$

$$\frac{15(3645.94)^2}{25} \leq \sigma^2$$

$$7,975,727.09 \leq \sigma^2$$

8-38 99% two-sided confidence interval on σ^2 for Izod impact test data
 $n = 20 \quad s = 0.25 \quad \chi_{0.005,19}^2 = 38.58 \quad \text{and} \quad \chi_{0.995,19}^2 = 6.84$

$$\frac{19(0.25)^2}{38.58} \leq \sigma^2 \leq \frac{19(0.25)^2}{6.84}$$

$$0.03078 \leq \sigma^2 \leq 0.1736$$

$$0.1754 < \sigma < 0.4167$$

8-39 95% confidence interval for σ : given $n = 51$, $s = 0.37$

First find the confidence interval for σ^2 :

For $\alpha = 0.05$ and $n = 51$, $\chi_{\alpha/2, n-1}^2 = \chi_{0.025, 50}^2 = 71.42$ and $\chi_{1-\alpha/2, n-1}^2 = \chi_{0.975, 50}^2 = 32.36$

$$\frac{50(0.37)^2}{71.42} \leq \sigma^2 \leq \frac{50(0.37)^2}{32.36}$$

$$0.096 \leq \sigma^2 \leq 0.2115$$

Taking the square root of the endpoints of this interval we obtain,

$$0.31 < \sigma < 0.46$$

8-40 99% two-sided confidence interval for σ for the hole diameter data. (Exercise 8-35)

For $\alpha = 0.01$ and $n = 15$, $\chi_{\alpha/2, n-1}^2 = \chi_{0.005, 14}^2 = 31.32$ and $\chi_{1-\alpha/2, n-1}^2 = \chi_{0.995, 14}^2 = 4.07$

$$\frac{14(0.008)^2}{31.32} \leq \sigma^2 \leq \frac{14(0.008)^2}{4.07}$$

$$0.00002861 \leq \sigma^2 \leq 0.0002201$$

$$0.005349 < \sigma < 0.01484$$

8-41 90% lower confidence bound on σ (the standard deviation the sugar content)

given $n = 10$, $s^2 = 23.04$

For $\alpha = 0.1$ and $n = 10$, $\chi_{\alpha, n-1}^2 = \chi_{0.1, 9}^2 = 14.68$

$$\frac{9(23.04)}{14.68} \leq \sigma^2$$

$$14.13 \leq \sigma^2$$

Take the square root of the endpoints of this interval to find the confidence interval for σ :

$$3.8 < \sigma$$

Section 8-5

8-42 95% Confidence Interval on the death rate from lung cancer.

$$\hat{p} = \frac{823}{1000} = 0.823 \quad n = 1000 \quad z_{\alpha/2} = 1.96$$

$$\hat{p} - z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq p \leq \hat{p} + z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.823 - 1.96 \sqrt{\frac{0.823(0.177)}{1000}} \leq p \leq 0.823 + 1.96 \sqrt{\frac{0.823(0.177)}{1000}}$$

$$0.7993 \leq p \leq 0.8467$$

8-43 $E = 0.03$, $\alpha = 0.05$, $z_{\alpha/2} = z_{0.025} = 1.96$ and $\hat{p} = 0.823$ as the initial estimate of p ,

$$n = \left(\frac{z_{\alpha/2}}{E} \right)^2 \hat{p}(1-\hat{p}) = \left(\frac{1.96}{0.03} \right)^2 0.823(1-0.823) = 621.79,$$

$n \cong 622$.

8-44 a.) 95% Confidence Interval on the true proportion of helmets showing damage.

$$\hat{p} = \frac{18}{50} = 0.36 \quad n = 50 \quad z_{\alpha/2} = 1.96$$

$$\hat{p} - z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq p \leq \hat{p} + z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.36 - 1.96 \sqrt{\frac{0.36(0.64)}{50}} \leq p \leq 0.36 + 1.96 \sqrt{\frac{0.36(0.64)}{50}}$$

$$0.227 \leq p \leq 0.493$$

$$b.) n = \left(\frac{z_{\alpha/2}}{E} \right)^2 p(1-p) = \left(\frac{1.96}{0.02} \right)^2 0.36(1-0.36) = 2212.76$$

$$n \cong 2213$$

$$c.) n = \left(\frac{z_{\alpha/2}}{E} \right)^2 p(1-p) = \left(\frac{1.96}{0.02} \right)^2 0.5(1-0.5) = 2401$$

8-45 The worst case would be for $p = 0.5$, thus with $E = 0.05$ and $\alpha = 0.01$, $z_{\alpha/2} = z_{0.005} = 2.58$ we obtain a sample size of:

$$n = \left(\frac{z_{\alpha/2}}{E} \right)^2 p(1-p) = \left(\frac{2.58}{0.05} \right)^2 0.5(1-0.5) = 665.64, \quad n \cong 666$$

8-46 99% Confidence Interval on the fraction defective.

$$\hat{p} = \frac{10}{800} = 0.0125 \quad n = 800 \quad z_{\alpha} = 2.33$$

$$\infty \leq p \leq \hat{p} + z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\infty \leq p \leq 0.0125 + 2.33 \sqrt{\frac{0.0125(0.9875)}{800}}$$

$$\infty \leq p \leq 0.0217$$

8-47 $E = 0.017$, $\alpha = 0.01$, $z_{\alpha/2} = z_{0.005} = 2.58$

$$n = \left(\frac{z_{\alpha/2}}{E} \right)^2 p(1-p) = \left(\frac{2.58}{0.017} \right)^2 0.5(1-0.5) = 5758.13, \quad n \cong 5759$$

8-48 95% Confidence Interval on the fraction defective produced with this tool.

$$\hat{p} = \frac{13}{300} = 0.04333 \quad n = 300 \quad z_{\alpha/2} = 1.96$$

$$\hat{p} - z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq p \leq \hat{p} + z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.04333 - 1.96 \sqrt{\frac{0.04333(0.95667)}{300}} \leq p \leq 0.04333 + 1.96 \sqrt{\frac{0.04333(0.95667)}{300}}$$

$$0.02029 \leq p \leq 0.06637$$

Section 8-6

8-49 95% prediction interval on the life of the next tire

given $\bar{x} = 60139.7$ $s = 3645.94$ $n = 16$

for $\alpha=0.05$ $t_{\alpha/2, n-1} = t_{0.025, 15} = 2.131$

$$\bar{x} - t_{0.025, 15} s \sqrt{1 + \frac{1}{n}} \leq x_{n+1} \leq \bar{x} + t_{0.025, 15} s \sqrt{1 + \frac{1}{n}}$$

$$60139.7 - 2.131(3645.94) \sqrt{1 + \frac{1}{16}} \leq x_{n+1} \leq 60139.7 + 2.131(3645.94) \sqrt{1 + \frac{1}{16}}$$

$$52131.1 \leq x_{n+1} \leq 68148.3$$

The prediction interval is considerably wider than the 95% confidence interval ($58,197.3 \leq \mu \leq 62,082.07$), which is to be expected since we are going beyond our data.

8-50 99% prediction interval on the Izod impact data

$n = 20$ $\bar{x} = 1.25$ $s = 0.25$ $t_{0.005, 19} = 2.861$

$$\bar{x} - t_{0.005, 19} s \sqrt{1 + \frac{1}{n}} \leq x_{n+1} \leq \bar{x} + t_{0.005, 19} s \sqrt{1 + \frac{1}{n}}$$

$$1.25 - 2.861(0.25) \sqrt{1 + \frac{1}{20}} \leq x_{n+1} \leq 1.25 + 2.861(0.25) \sqrt{1 + \frac{1}{20}}$$

$$0.517 \leq x_{n+1} \leq 1.983$$

The lower bound of the 99% prediction interval is considerably lower than the 99% confidence interval ($1.108 \leq \mu \leq \infty$), which is to be expected since we are going beyond our data.

8-51 Given $\bar{x} = 317.2$ $s = 15.7$ $n = 10$ for $\alpha=0.05$ $t_{\alpha/2, n-1} = t_{0.005, 9} = 3.250$

$$\begin{aligned} \bar{x} - t_{0.005,9} s \sqrt{1 + \frac{1}{n}} &\leq x_{n+1} \leq \bar{x} + t_{0.005,9} s \sqrt{1 + \frac{1}{n}} \\ 317.2 - 3.250(15.7) \sqrt{1 + \frac{1}{10}} &\leq x_{n+1} \leq 317.2 + 3.250(15.7) \sqrt{1 + \frac{1}{10}} \\ 263.7 &\leq x_{n+1} \leq 370.7 \end{aligned}$$

The length of the prediction interval is longer.

- 8-52 99% prediction interval on the polyunsaturated fat
 $n = 6$ $\bar{x} = 16.98$ $s = 0.319$ $t_{0.005,5} = 4.032$

$$\begin{aligned} \bar{x} - t_{0.005,5} s \sqrt{1 + \frac{1}{n}} &\leq x_{n+1} \leq \bar{x} + t_{0.005,5} s \sqrt{1 + \frac{1}{n}} \\ 16.98 - 4.032(0.319) \sqrt{1 + \frac{1}{6}} &\leq x_{n+1} \leq 16.98 + 4.032(0.319) \sqrt{1 + \frac{1}{6}} \\ 15.59 &\leq x_{n+1} \leq 18.37 \end{aligned}$$

The length of the prediction interval is a lot longer than the width of the confidence interval
 $16.455 \leq \mu \leq 17.505$.

- 8-53 90% prediction interval on the next specimen of concrete tested
given $\bar{x} = 2260$ $s = 35.57$ $n = 12$ for $\alpha = 0.05$ and $n = 12$, $t_{\alpha/2, n-1} = t_{0.05, 11} = 1.796$

$$\begin{aligned} \bar{x} - t_{0.05, 11} s \sqrt{1 + \frac{1}{n}} &\leq x_{n+1} \leq \bar{x} + t_{0.05, 11} s \sqrt{1 + \frac{1}{n}} \\ 2260 - 1.796(35.57) \sqrt{1 + \frac{1}{12}} &\leq x_{n+1} \leq 2260 + 1.796(35.57) \sqrt{1 + \frac{1}{12}} \\ 2193.5 &\leq x_{n+1} \leq 2326.5 \end{aligned}$$

- 8-54 95% prediction interval on the next rod diameter tested
 $n = 15$ $\bar{x} = 8.23$ $s = 0.025$ $t_{0.025, 14} = 2.145$

$$\begin{aligned} \bar{x} - t_{0.025, 14} s \sqrt{1 + \frac{1}{n}} &\leq x_{n+1} \leq \bar{x} + t_{0.025, 14} s \sqrt{1 + \frac{1}{n}} \\ 8.23 - 2.145(0.025) \sqrt{1 + \frac{1}{15}} &\leq x_{n+1} \leq 8.23 + 2.145(0.025) \sqrt{1 + \frac{1}{15}} \\ 8.17 &\leq x_{n+1} \leq 8.29 \end{aligned}$$

95% two-sided confidence interval on mean rod diameter is $8.216 \leq \mu \leq 8.244$

- 8-55 90% prediction interval on wall thickness on the next bottle tested.

given $\bar{x} = 4.05$ $s = 0.08$ $n = 25$ for $t_{\alpha/2, n-1} = t_{0.05, 24} = 1.711$

$$\bar{x} - t_{0.05, 24} s \sqrt{1 + \frac{1}{n}} \leq x_{n+1} \leq \bar{x} + t_{0.05, 24} s \sqrt{1 + \frac{1}{n}}$$

$$4.05 - 1.711(0.08) \sqrt{1 + \frac{1}{25}} \leq x_{n+1} \leq 4.05 + 1.711(0.08) \sqrt{1 + \frac{1}{25}}$$

$$3.91 \leq x_{n+1} \leq 4.19$$

- 8-56 To obtain a one sided prediction interval, use $t_{\alpha, n-1}$ instead of $t_{\alpha/2, n-1}$
 Since we want a 95% one sided prediction interval, $t_{\alpha/2, n-1} = t_{0.05, 24} = 1.711$
 and $\bar{x} = 4.05$ $s = 0.08$ $n = 25$

$$\bar{x} - t_{0.05, 24} s \sqrt{1 + \frac{1}{n}} \leq x_{n+1}$$

$$4.05 - 1.711(0.08) \sqrt{1 + \frac{1}{25}} \leq x_{n+1}$$

$$3.91 \leq x_{n+1}$$

The prediction interval bound is a lot lower than the confidence interval bound of 4.023 mm

- 8-57 99% prediction interval for enrichment data given $\bar{x} = 2.9$ $s = 0.099$ $n = 12$ for $\alpha = 0.01$ and $n = 12$, $t_{\alpha/2, n-1} = t_{0.005, 11} = 3.106$

$$\bar{x} - t_{0.005, 12} s \sqrt{1 + \frac{1}{n}} \leq x_{n+1} \leq \bar{x} + t_{0.005, 12} s \sqrt{1 + \frac{1}{n}}$$

$$2.9 - 3.106(0.099) \sqrt{1 + \frac{1}{12}} \leq x_{n+1} \leq 2.9 + 3.106(0.099) \sqrt{1 + \frac{1}{12}}$$

$$2.58 \leq x_{n+1} \leq 3.22$$

The prediction interval is much wider than the 99% CI on the population mean ($2.813 \leq \mu \leq 2.991$).

- 8-58 95% Prediction Interval on the volume of syrup of the next beverage dispensed
 $\bar{x} = 1.10$ $s = 0.015$ $n = 25$ $t_{\alpha/2, n-1} = t_{0.025, 24} = 2.064$

$$\bar{x} - t_{0.025, 24} s \sqrt{1 + \frac{1}{n}} \leq x_{n+1} \leq \bar{x} + t_{0.025, 24} s \sqrt{1 + \frac{1}{n}}$$

$$1.10 - 2.064(0.015) \sqrt{1 + \frac{1}{25}} \leq x_{n+1} \leq 1.10 + 2.064(0.015) \sqrt{1 + \frac{1}{25}}$$

$$1.068 \leq x_{n+1} \leq 1.13$$

The prediction interval is wider than the confidence interval: $1.093 \leq \mu \leq 1.106$

- 8-59 90% prediction interval the value of the natural frequency of the next beam of this type that will be tested. given $\bar{x} = 231.67$, $s=1.53$ For $\alpha = 0.10$ and $n = 5$, $t_{\alpha/2, n-1} = t_{0.05, 4} = 2.132$

$$\bar{x} - t_{0.05, 4} s \sqrt{1 + \frac{1}{n}} \leq x_{n+1} \leq \bar{x} + t_{0.05, 4} s \sqrt{1 + \frac{1}{n}}$$

$$231.67 - 2.132(1.53) \sqrt{1 + \frac{1}{5}} \leq x_{n+1} \leq 231.67 + 2.132(1.53) \sqrt{1 + \frac{1}{5}}$$

$$228.1 \leq x_{n+1} \leq 235.2$$

The 90% prediction in interval is greater than the 90% CI.

Section 8-7

- 8-60 95% tolerance interval on the life of the tires that has a 95% CL
given $\bar{x} = 60139.7$ $s = 3645.94$ $n = 16$ we find $k=2.903$

$$\bar{x} - ks, \bar{x} + ks$$

$$60139.7 - 2.903(3645.94), 60139.7 + 2.903(3645.94)$$

$$(49555.54, 70723.86)$$

95% confidence interval ($58,197.3 \leq \mu \leq 62,082.07$) is shorter than the 95% tolerance interval.

- 8-61 99% tolerance interval on the Izod impact strength PVC pipe that has a 90% CL
given $\bar{x}=1.25$, $s=0.25$ and $n=20$ we find $k=3.368$

$$\bar{x} - ks, \bar{x} + ks$$

$$1.25 - 3.368(0.25), 1.25 + 3.368(0.25)$$

$$(0.408, 2.092)$$

The 99% tolerance interval is much wider than the 99% confidence interval on the population mean ($1.090 \leq \mu \leq 1.410$).

- 8-62 99% tolerance interval on the brightness of television tubes that has a 95% CL
given $\bar{x} = 317.2$ $s = 15.7$ $n = 10$ we find $k=4.433$

$$\bar{x} - ks, \bar{x} + ks$$

$$317.2 - 4.433(15.7), 317.2 + 4.433(15.7)$$

$$(247.60, 386.80)$$

The 99% tolerance interval is much wider than the 95% confidence interval on the population mean $301.06 \leq \mu \leq 333.34$.

- 8-63 99% tolerance interval on the polyunsaturated fatty acid in this type of margarine that has a confidence level of 95% $\bar{x} = 16.98$ $s = 0.319$ $n=6$ and $k = 5.775$

$$\bar{x} - ks, \bar{x} + ks$$

$$16.98 - 5.775(0.319), 16.98 + 5.775(0.319)$$

$$(15.14, 18.82)$$

The 99% tolerance interval is much wider than the 99% confidence interval on the population mean ($16.46 \leq \mu \leq 17.51$).

- 8-64 90% tolerance interval on the comprehensive strength of concrete that has a 90% CL given $\bar{x} = 2260$ $s = 35.57$ $n = 12$ we find $k=2.404$

$$\bar{x} - ks, \bar{x} + ks$$

$$2260 - 2.404(35.57), 2260 + 2.404(35.57)$$

$$(2174.5, 2345.5)$$

The 90% tolerance interval is much wider than the 95% confidence interval on the population mean $2237.3 \leq \mu \leq 2282.5$.

- 8-65 95% tolerance interval on the diameter of the rods in exercise 8-27 that has a 90% confidence level. $\bar{x} = 8.23$ $s = 0.025$ $n=15$ and $k=2.713$

$$\bar{x} - ks, \bar{x} + ks$$

$$8.23 - 2.713(0.025), 8.23 + 2.713(0.025)$$

$$(8.16, 8.30)$$

The 95% tolerance interval is wider than the 95% confidence interval on the population mean ($8.216 \leq \mu \leq 8.244$).

- 8-66 90% tolerance interval on wall thickness measurements that have a 90% CL given $\bar{x} = 4.05$ $s = 0.08$ $n = 25$ we find $k=2.077$

$$\bar{x} - ks, \bar{x} + ks$$

$$4.05 - 2.077(0.08), 4.05 + 2.077(0.08)$$

$$(3.88, 4.22)$$

The lower bound of the 90% tolerance interval is much lower than the lower bound on the 95% confidence interval on the population mean ($4.023 \leq \mu \leq \infty$)

- 8-67 90% lower tolerance bound on bottle wall thickness that has confidence level 90%. given $\bar{x} = 4.05$ $s = 0.08$ $n = 25$ and $k = 1.702$

$$\bar{x} - ks$$

$$4.05 - 1.702(0.08)$$

$$3.91$$

The lower tolerance bound is of interest if we want to make sure the wall thickness is at least a certain value so that the bottle will not break.

- 8-68 99% tolerance interval on rod enrichment data that have a 95% CL

given $\bar{x} = 2.9$ $s = 0.099$ $n = 12$ we find $k=4.150$

$$\begin{aligned} & \bar{x} - ks, \bar{x} + ks \\ & 2.9 - 4.150(0.099), 2.9 + 4.150(0.099) \\ & (2.49, 3.31) \end{aligned}$$

The 99% tolerance interval is much wider than the 95% CI on the population mean ($2.84 \leq \mu \leq 2.96$).

8-69 95% tolerance interval on the syrup volume that has 90% confidence level
 $\bar{x} = 1.10$ $s = 0.015$ $n = 25$ and $k=2.474$

$$\begin{aligned} & \bar{x} - ks, \bar{x} + ks \\ & 1.10 - 2.474(0.015), 1.10 + 2.474(0.015) \\ & (1.06, 1.14) \end{aligned}$$

Supplemental Exercises

8-70 Where $\alpha_1 + \alpha_2 = \alpha$. Let $\alpha = 0.05$

Interval for $\alpha_1 = \alpha_2 = \alpha/2 = 0.025$

The confidence level for $\bar{x} - 1.96\sigma/\sqrt{n} \leq \mu \leq \bar{x} + 1.96\sigma/\sqrt{n}$ is determined by the

by the value of z_0 which is 1.96. From Table II, we find $\Phi(1.96) = P(Z < 1.96) = 0.975$ and the confidence level is 95%.

Interval for $\alpha_1 = 0.01$, $\alpha_2 = 0.04$

The confidence interval is $\bar{x} - 2.33\sigma/\sqrt{n} \leq \mu \leq \bar{x} + 1.75\sigma/\sqrt{n}$, the confidence level is the same since $\alpha = 0.05$. The symmetric interval does not affect the level of significance.

8-71 $\mu = 50$ σ unknown
a) $n = 16$ $\bar{x} = 52$ $s = 1.5$

$$t_o = \frac{52 - 50}{8/\sqrt{16}} = 1$$

The P-value for $t_0 = 1$, degrees of freedom = 15, is between 0.1 and 0.25. Thus we would conclude that the results are not very unusual.

b) $n = 30$

$$t_o = \frac{52 - 50}{8/\sqrt{30}} = 1.37$$

The P-value for $t_0 = 1.37$, degrees of freedom = 29, is between 0.05 and 0.1. Thus we would conclude that the results are somewhat unusual.

c) $n = 100$ (with $n > 30$, the standard normal table can be used for this problem)

$$z_o = \frac{52 - 50}{8/\sqrt{100}} = 2.5$$

The P-value for $z_0 = 2.5$, is 0.00621. Thus we would conclude that the results are very unusual.

d) For constant values of \bar{x} and s , increasing only the sample size, we see that the standard error of \bar{X} decreases and consequently a sample mean value of 52 when the true mean is 50 is more unusual for the larger sample sizes.

8-72 $\mu = 50$, $\sigma^2 = 5$

a.) For $n = 16$ find $P(s^2 \geq 7.44)$ or $P(s^2 \leq 2.56)$

$$P(s^2 \geq 7.44) = P\left(\chi_{15}^2 \geq \frac{15(7.44)}{5}\right) = 0.05 \leq P(\chi_{15}^2 \geq 22.32) \leq 0.10$$

Using Minitab $P(s^2 \geq 7.44) = 0.0997$

$$P(s^2 \leq 2.56) = P\left(\chi_{15}^2 \leq \frac{15(2.56)}{5}\right) = 0.05 \leq P(\chi_{15}^2 \leq 7.68) \leq 0.10$$

Using Minitab $P(s^2 \leq 2.56) = 0.064$

b) For $n = 30$ find $P(s^2 \geq 7.44)$ or $P(s^2 \leq 2.56)$

$$P(s^2 \geq 7.44) = P\left(\chi_{29}^2 \geq \frac{29(7.44)}{5}\right) = 0.025 \leq P(\chi_{29}^2 \geq 43.15) \leq 0.05$$

Using Minitab $P(s^2 \geq 7.44) = 0.044$

$$P(s^2 \leq 2.56) = P\left(\chi_{29}^2 \leq \frac{29(2.56)}{5}\right) = 0.01 \leq P(\chi_{29}^2 \leq 14.85) \leq 0.025$$

Using Minitab $P(s^2 \leq 2.56) = 0.014$.

c) For $n = 71$ $P(s^2 \geq 7.44)$ or $P(s^2 \leq 2.56)$

$$P(s^2 \geq 7.44) = P\left(\chi_{70}^2 \geq \frac{70(7.44)}{5}\right) = 0.005 \leq P(\chi_{70}^2 \geq 104.16) \leq 0.01$$

Using Minitab $P(s^2 \geq 7.44) = 0.0051$

$$P(s^2 \leq 2.56) = P\left(\chi_{70}^2 \leq \frac{70(2.56)}{5}\right) = P(\chi_{70}^2 \leq 35.84) \leq 0.005$$

Using Minitab $P(s^2 \leq 2.56) < 0.001$

d) The probabilities get smaller as n increases. As n increases, the sample variance should approach the population variance; therefore, the likelihood of obtaining a sample variance much larger than the population variance will decrease.

e) The probabilities get smaller as n increases. As n increases, the sample variance should approach the population variance; therefore, the likelihood of obtaining a sample variance much smaller than the population variance will decrease.

- 8-73 a) The data appear to follow a normal distribution based on the normal probability plot since the data fall along a straight line.
 b) It is important to check for normality of the distribution underlying the sample data since the confidence intervals to be constructed should have the assumption of normality for the results to be reliable (especially since the sample size is less than 30 and the central limit theorem does not apply).
 c) No, with 95% confidence, we can not infer that the true mean could be 14.05 since this value is not contained within the given 95% confidence interval.
 d) As with part b, to construct a confidence interval on the variance, the normality assumption must hold for the results to be reliable.
 e) Yes, it is reasonable to infer that the variance could be 0.35 since the 95% confidence interval on the variance contains this value.
 f) i) & ii) No, doctors and children would represent two completely different populations not represented by the population of Canadian Olympic hockey players. Since doctors nor children were the target of this study or part of the sample taken, the results should not be extended to these groups.

- 8-74 a.) The probability plot shows that the data appear to be normally distributed. Therefore, there is no evidence conclude that the comprehensive strength data are normally distributed.
 b.) 99% lower confidence bound on the mean $\bar{x} = 25.12$, $s = 8.42$, $n = 9$ $t_{0.01,8} = 2.896$

$$\bar{x} - t_{0.01,8} \left(\frac{s}{\sqrt{n}} \right) \leq \mu$$

$$25.12 - 2.896 \left(\frac{8.42}{\sqrt{9}} \right) \leq \mu$$

$$16.99 \leq \mu$$

The lower bound on the 99% confidence interval shows that the mean comprehensive strength will most likely be greater than 16.99 Megapascals.

- c.) 98% lower confidence bound on the mean $\bar{x} = 25.12$, $s = 8.42$, $n = 9$ $t_{0.01,8} = 2.896$

$$\bar{x} - t_{0.01,8} \left(\frac{s}{\sqrt{n}} \right) \leq \mu \leq \bar{x} - t_{0.01,8} \left(\frac{s}{\sqrt{n}} \right)$$

$$25.12 - 2.896 \left(\frac{8.42}{\sqrt{9}} \right) \leq \mu \leq 25.12 - 2.896 \left(\frac{8.42}{\sqrt{9}} \right)$$

$$16.99 \leq \mu \leq 33.25$$

The bounds on the 98% two-sided confidence interval shows that the mean comprehensive strength will most likely be greater than 16.99 Megapascals and less than 33.25 Megapascals. The lower bound of the 99% one sided CI is the same as the lower bound of the 98% two-sided CI (this is because of the value of α)

- d.) 99% one-sided upper bound on the confidence interval on σ^2 comprehensive strength

$$s = 8.42, \quad s^2 = 70.90 \quad \chi_{0.99,8}^2 = 1.65$$

$$\sigma^2 \leq \frac{8(8.42)^2}{1.65}$$

$$\sigma^2 \leq 343.74$$

The upper bound on the 99% confidence interval on the variance shows that the variance of the comprehensive strength will most likely be less than 343.74 Megapascals².

- e.) 98% one-sided upper bound on the confidence interval on σ^2 comprehensive strength

$$s = 8.42, \quad s^2 = 70.90 \quad \chi_{0.01,9}^2 = 20.09 \quad \chi_{0.99,8}^2 = 1.65$$

$$\frac{8(8.42)^2}{20.09} \leq \sigma^2 \leq \frac{8(8.42)^2}{1.65}$$

$$28.23 \leq \sigma^2 \leq 343.74$$

The bounds on the 98% two-sided confidence-interval on the variance shows that the variance of the comprehensive strength will most likely be less than 343.74 Megapascals² and greater than 28.23 Megapascals². The upper bound of the 99% one-sided CI is the same as the upper bound of the 98% two-sided CI (this is because of the value of α)

f.) 98% lower confidence bound on the mean $\bar{x} = 23, \quad s = 6.07, \quad n = 9 \quad t_{0.01,8} = 2.896$

$$\bar{x} - t_{0.01,8} \left(\frac{s}{\sqrt{n}} \right) \leq \mu \leq \bar{x} - t_{0.01,8} \left(\frac{s}{\sqrt{n}} \right)$$

$$23 - 2.896 \left(\frac{6.07}{\sqrt{9}} \right) \leq \mu \leq 23 - 2.896 \left(\frac{6.07}{\sqrt{9}} \right)$$

$$17.14 \leq \mu \leq 28.86$$

98% one-sided upper bound on the confidence interval on σ^2 comprehensive strength

$$s = 6.07, \quad s^2 = 36.9 \quad \chi_{0.01,9}^2 = 20.09 \quad \chi_{0.99,8}^2 = 1.65$$

$$\frac{8(6.07)^2}{20.09} \leq \sigma^2 \leq \frac{8(6.07)^2}{1.65}$$

$$14.67 \leq \sigma^2 \leq 178.64$$

Fixing the mistake decreased the values of the sample mean and the sample standard deviation. Since the sample standard deviation was decreased. The width of the confidence intervals were also decreased.

g.) 98% lower confidence bound on the mean $\bar{x} = 25, \quad s = 8.41, \quad n = 9 \quad t_{0.01,8} = 2.896$

$$\bar{x} - t_{0.01,8} \left(\frac{s}{\sqrt{n}} \right) \leq \mu \leq \bar{x} - t_{0.01,8} \left(\frac{s}{\sqrt{n}} \right)$$

$$25 - 2.896 \left(\frac{8.41}{\sqrt{9}} \right) \leq \mu \leq 25 - 2.896 \left(\frac{8.41}{\sqrt{9}} \right)$$

$$16.88 \leq \mu \leq 33.12$$

98% one-sided upper bound on the confidence interval on σ^2 comprehensive strength

$$s = 8.41, \quad s^2 = 70.73 \quad \chi_{0.01,9}^2 = 20.09 \quad \chi_{0.99,8}^2 = 1.65$$

$$\frac{8(8.41)^2}{20.09} \leq \sigma^2 \leq \frac{8(8.41)^2}{1.65}$$

$$28.16 \leq \sigma^2 \leq 342.94$$

Fixing the mistake did not have an affect on the sample mean or the sample standard deviation. They are very close to the original values. The width of the confidence intervals are also very similar.

h.) When a mistaken value is near the sample mean, the mistake will not affect the sample mean, standard deviation or confidence intervals greatly. However, when the mistake is not near the sample mean, the

value can greatly affect the sample mean, standard deviation and confidence intervals. The farther from the mean, the greater the effect.

8-75 With $\sigma = 8$, the 95% confidence interval on the mean has length of at most 5; the error is then $E = 2.5$.

$$a) n = \left(\frac{z_{0.025}}{2.5} \right)^2 8^2 = \left(\frac{1.96}{2.5} \right)^2 64 = 39.34 = 40$$

$$b) n = \left(\frac{z_{0.025}}{2.5} \right)^2 6^2 = \left(\frac{1.96}{2.5} \right)^2 36 = 22.13 = 23$$

As the standard deviation decreases, with all other values held constant, the sample size necessary to maintain the acceptable level of confidence and the length of the interval, decreases.

8-76 $\bar{x} = 15.33$ $s = 0.62$ $n = 20$ $k = 2.564$

a.) 95% Tolerance Interval of hemoglobin values with 90% confidence

$$\begin{aligned} & \bar{x} - ks, \bar{x} + ks \\ & 15.33 - 2.564(0.62), 15.33 + 2.564(0.62) \\ & (13.74, 16.92) \end{aligned}$$

b.) 99% Tolerance Interval of hemoglobin values with 90% confidence $k = 3.368$

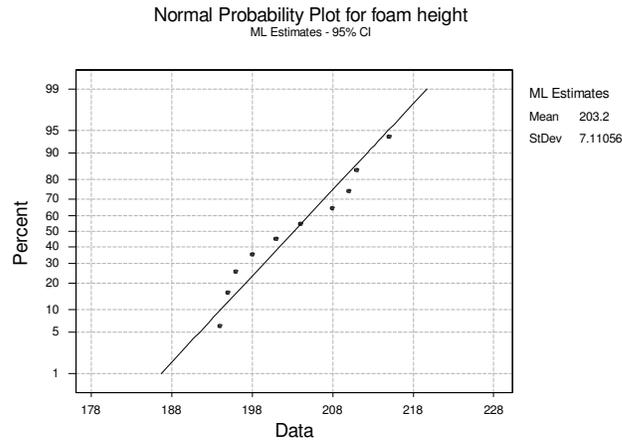
$$\begin{aligned} & \bar{x} - ks, \bar{x} + ks \\ & 15.33 - 3.368(0.62), 15.33 + 3.368(0.62) \\ & (13.24, 17.42) \end{aligned}$$

8-77 95% prediction interval for the next sample of concrete that will be tested.

given $\bar{x} = 25.12$ $s = 8.42$ $n = 9$ for $\alpha = 0.05$ and $n = 9$, $t_{\alpha/2, n-1} = t_{0.025, 8} = 2.306$

$$\begin{aligned} & \bar{x} - t_{0.025, 8} s \sqrt{1 + \frac{1}{n}} \leq x_{n+1} \leq \bar{x} + t_{0.025, 8} s \sqrt{1 + \frac{1}{n}} \\ & 25.12 - 2.306(8.42) \sqrt{1 + \frac{1}{9}} \leq x_{n+1} \leq 25.12 + 2.306(8.42) \sqrt{1 + \frac{1}{9}} \\ & 4.65 \leq x_{n+1} \leq 45.59 \end{aligned}$$

- 8-78 a.) There is no evidence to reject the assumption that the data are normally distributed.



- b.) 95% confidence interval on the mean $\bar{x} = 203.20$, $s = 7.5$, $n = 10$ $t_{0.025,9} = 2.262$

$$\bar{x} - t_{0.025,9} \left(\frac{s}{\sqrt{n}} \right) \leq \mu \leq \bar{x} + t_{0.025,9} \left(\frac{s}{\sqrt{n}} \right)$$

$$203.2 - 2.262 \left(\frac{7.50}{\sqrt{10}} \right) \leq \mu \leq 203.2 + 2.262 \left(\frac{7.50}{\sqrt{10}} \right)$$

$$197.84 \leq \mu \leq 208.56$$

- c.) 95% prediction interval on a future sample

$$\bar{x} - t_{0.025,9} s \sqrt{1 + \frac{1}{n}} \leq \mu \leq \bar{x} + t_{0.025,9} s \sqrt{1 + \frac{1}{n}}$$

$$203.2 - 2.262(7.50) \sqrt{1 + \frac{1}{10}} \leq \mu \leq 203.2 + 2.262(7.50) \sqrt{1 + \frac{1}{10}}$$

$$185.41 \leq \mu \leq 220.99$$

- d.) 95% tolerance interval on foam height with 99% confidence $k = 4.265$

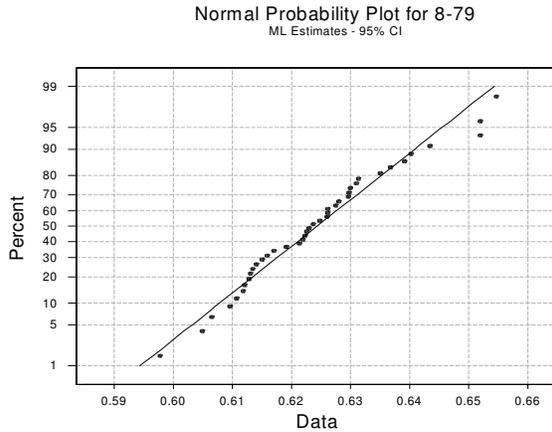
$$\bar{x} - ks, \bar{x} + ks$$

$$203.2 - 4.265(7.5), 203.2 + 4.265(7.5)$$

$$(171.21, 235.19)$$

- e.) The 95% CI on the population mean has the smallest interval. This type of interval tells us that 95% of such intervals would contain the population mean. The 95% prediction interval, tell us where, most likely, the next data point will fall. This interval is quite a bit larger than the CI on the mean. The tolerance interval is the largest interval of all. It tells us the limits that will include 95% of the data with 99% confidence.

8-79 a) Normal probability plot for the coefficient of restitution



b.) 99% CI on the true mean coefficient of restitution

$$\bar{x} = 0.624, s = 0.013, n = 40 \quad t_{\alpha/2, n-1} = t_{0.005, 39} = 2.7079$$

$$\bar{x} - t_{0.005, 39} \frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + t_{0.005, 39} \frac{s}{\sqrt{n}}$$

$$0.624 - 2.7079 \frac{0.013}{\sqrt{40}} \leq \mu \leq 0.624 + 2.7079 \frac{0.013}{\sqrt{40}}$$

$$0.618 \leq \mu \leq 0.630$$

c.) 99% prediction interval on the coefficient of restitution for the next baseball that will be tested.

$$\bar{x} - t_{0.005, 39} s \sqrt{1 + \frac{1}{n}} \leq x_{n+1} \leq \bar{x} + t_{0.005, 39} s \sqrt{1 + \frac{1}{n}}$$

$$0.624 - 2.7079(0.013) \sqrt{1 + \frac{1}{40}} \leq x_{n+1} \leq 0.624 + 2.7079(0.013) \sqrt{1 + \frac{1}{40}}$$

$$0.588 \leq x_{n+1} \leq 0.660$$

d.) 99% tolerance interval on the coefficient of restitution with a 95% level of confidence

$$(\bar{x} - ks, \bar{x} + ks)$$

$$(0.624 - 3.213(0.013), 0.624 + 3.213(0.013))$$

$$(0.582, 0.666)$$

e.) The confidence interval in part (b) describes the confidence interval on the population mean and we may interpret this to mean that 99% of such intervals will cover the population mean. The prediction interval tells us that within a 99% probability that the next baseball will have a coefficient of restitution between 0.588 and 0.660. And the tolerance interval captures 99% of the values of the normal distribution with a 95% level of confidence.

8-80 95% Confidence Interval on the death rate from lung cancer.

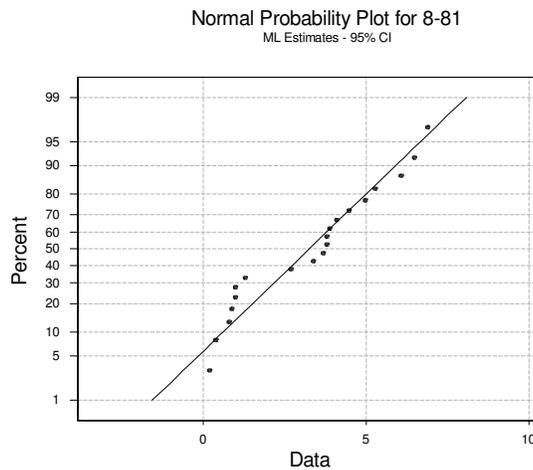
$$\hat{p} = \frac{8}{40} = 0.2 \quad n = 40 \quad z_{\alpha} = 1.65$$

$$\hat{p} - z_{\alpha} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq p$$

$$0.2 - 1.65 \sqrt{\frac{0.2(0.8)}{40}} \leq p$$

$$0.0956 \leq p$$

- 8-81 a.) The normal probability shows that the data are mostly follow the straight line, however, there are some points that deviate from the line near the middle. It is probably safe to assume that the data are normal.



- b.) 95% CI on the mean dissolved oxygen concentration

$$\bar{x} = 3.265, s = 2.127, n = 20 \quad t_{\alpha/2, n-1} = t_{0.025, 19} = 2.093$$

$$\bar{x} - t_{0.025, 19} \frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + t_{0.025, 19} \frac{s}{\sqrt{n}}$$

$$3.265 - 2.093 \frac{2.127}{\sqrt{20}} \leq \mu \leq 3.265 + 2.093 \frac{2.127}{\sqrt{20}}$$

$$2.270 \leq \mu \leq 4.260$$

- c.) 95% prediction interval on the oxygen concentration for the next stream in the system that will be tested..

$$\bar{x} - t_{0.025, 19} s \sqrt{1 + \frac{1}{n}} \leq x_{n+1} \leq \bar{x} + t_{0.025, 19} s \sqrt{1 + \frac{1}{n}}$$

$$3.265 - 2.093(2.127) \sqrt{1 + \frac{1}{20}} \leq x_{n+1} \leq 3.265 + 2.093(2.127) \sqrt{1 + \frac{1}{20}}$$

$$-1.297 \leq x_{n+1} \leq 7.827$$

- d.) 95% tolerance interval on the values of the dissolved oxygen concentration with a 99% level of confidence

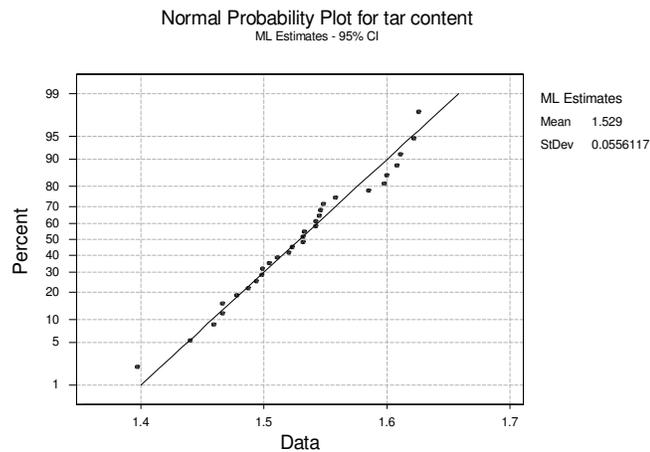
$$(\bar{x} - ks, \bar{x} + ks)$$

$$(3.265 - 3.168(2.127), 3.265 + 3.168(2.127))$$

$$(-3.473, 10.003)$$

e.) The confidence interval in part (b) describes the confidence interval on the population mean and we may interpret this to mean that there is a 95% probability that the interval may cover the population mean. The prediction interval tells us that within that within a 95% probability that the next stream will have an oxygen concentration between -1.297 and 7.827mg/L. And the tolerance interval captures 95% of the values of the normal distribution with a 99% confidence level.

- 8-82 a.) There is no evidence to support that the data are not normally distributed. The data points appear to fall along the normal probability line.



- b.) 99% CI on the mean tar content

$$\bar{x} = 1.529, s = 0.0566, n = 30 \quad t_{\alpha/2, n-1} = t_{0.005, 29} = 2.756$$

$$\bar{x} - t_{0.005, 29} \frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + t_{0.005, 29} \frac{s}{\sqrt{n}}$$

$$1.529 - 2.756 \frac{0.0566}{\sqrt{30}} \leq \mu \leq 1.529 + 2.756 \frac{0.0566}{\sqrt{30}}$$

$$1.501 \leq \mu \leq 1.557$$

- e.) 99% prediction interval on the tar content for the next sample that will be tested..

$$\bar{x} - t_{0.005, 19} s \sqrt{1 + \frac{1}{n}} \leq x_{n+1} \leq \bar{x} + t_{0.005, 19} s \sqrt{1 + \frac{1}{n}}$$

$$1.529 - 2.756(0.0566) \sqrt{1 + \frac{1}{30}} \leq x_{n+1} \leq 1.529 + 2.756(0.0566) \sqrt{1 + \frac{1}{30}}$$

$$1.370 \leq x_{n+1} \leq 1.688$$

f.) 99% tolerance interval on the values of the tar content with a 95% level of confidence

$$(\bar{x} - ks, \bar{x} + ks)$$

$$(1.529 - 3.350(0.0566), 1.529 + 3.350(0.0566))$$

$$(1.339, 1.719)$$

e.) The confidence interval in part (b) describes the confidence interval on the population mean and we may interpret this to mean that 95% of such intervals will cover the population mean. The prediction interval tells us that within a 95% probability that the sample will have a tar content between 1.370 and 1.688. And the tolerance interval captures 95% of the values of the normal distribution with a 99% confidence level.

8-83 a.) 95% Confidence Interval on the population proportion

$$n=1200 \quad x=8 \quad \hat{p} = 0.0067 \quad z_{\alpha/2} = z_{0.025} = 1.96$$

$$\hat{p} - z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq p \leq \hat{p} + z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.0067 - 1.96 \sqrt{\frac{0.0067(1-0.0067)}{1200}} \leq p \leq 0.0067 + 1.96 \sqrt{\frac{0.0067(1-0.0067)}{1200}}$$

$$0.0021 \leq p \leq 0.0113$$

b) No, there is not evidence to support the claim that the fraction of defective units produced is one percent or less. This is because the upper limit of the control limit is greater than 0.01.

8-84 a.) 99% Confidence Interval on the population proportion

$$n=1600 \quad x=8 \quad \hat{p} = 0.005 \quad z_{\alpha/2} = z_{0.005} = 2.58$$

$$\hat{p} - z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq p \leq \hat{p} + z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.005 - 2.58 \sqrt{\frac{0.005(1-0.005)}{1600}} \leq p \leq 0.005 + 2.58 \sqrt{\frac{0.005(1-0.005)}{1600}}$$

$$0.0004505 \leq p \leq 0.009549$$

b.) $E = 0.008, \alpha = 0.01, z_{\alpha/2} = z_{0.005} = 2.58$

$$n = \left(\frac{z_{\alpha/2}}{E} \right)^2 p(1-p) = \left(\frac{2.58}{0.008} \right)^2 0.005(1-0.005) = 517.43, \quad n \cong 518$$

c.) $E = 0.008, \alpha = 0.01, z_{\alpha/2} = z_{0.005} = 2.58$

$$n = \left(\frac{z_{\alpha/2}}{E} \right)^2 p(1-p) = \left(\frac{2.58}{0.008} \right)^2 0.5(1-0.5) = 26001.56, \quad n \cong 26002$$

d.) Knowing an estimate of the population proportion reduces the required sample size by a significant amount. A sample size of 518 is much more reasonable than a sample size of over 26,000.

$$8-85 \quad \hat{p} = \frac{117}{484} = 0.242$$

a) 90% confidence interval; $z_{\alpha/2} = 1.645$

$$\hat{p} - z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq p \leq \hat{p} + z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.210 \leq p \leq 0.274$$

With 90% confidence, we believe the true proportion of new engineering graduates who were planning to continue studying for an advanced degree lies between 0.210 and 0.274.

b) 95% confidence interval; $z_{\alpha/2} = 1.96$

$$\hat{p} - z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq p \leq \hat{p} + z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.204 \leq p \leq 0.280$$

With 95% confidence, we believe the true proportion of new engineering graduates who were planning to continue studying for an advanced degree lies between 0.204 and 0.280.

c) Comparison of parts a and b:

The 95% confidence interval is larger than the 90% confidence interval. Higher confidence always yields larger intervals, all other values held constant.

d) Yes, since both intervals contain the value 0.25, thus there is not enough evidence to determine that the true proportion is not actually 0.25.

Mind Expanding Exercises

$$8-86 \quad \text{a.) } P(\chi_{1-\frac{\alpha}{2}, 2r}^2 < 2\lambda T_r < \chi_{\frac{\alpha}{2}, 2r}^2) = 1 - \alpha$$

$$= P\left(\frac{\chi_{1-\frac{\alpha}{2}, 2r}^2}{2T_r} < \lambda < \frac{\chi_{\frac{\alpha}{2}, 2r}^2}{2T_r}\right)$$

$$\text{Then a confidence interval for } \mu = \frac{1}{\lambda} \text{ is } \left(\frac{2T_r}{\chi_{\frac{\alpha}{2}, 2r}^2}, \frac{2T_r}{\chi_{1-\frac{\alpha}{2}, 2r}^2}\right)$$

b) $n = 20$, $r = 10$, and the observed value of T_r is $199 + 10(29) = 489$.

$$\text{A 95\% confidence interval for } \frac{1}{\lambda} \text{ is } \left(\frac{2(489)}{34.17}, \frac{2(489)}{9.59}\right) = (28.62, 101.98)$$

$$8-87 \quad \alpha_1 = \int_{z_{\alpha_1}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = 1 - \int_{-\infty}^{z_{\alpha_1}} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$$

Therefore, $1 - \alpha_1 = \Phi(z_{\alpha_1})$.

To minimize L we need to minimize $\Phi^{-1}(1 - \alpha_1) + \Phi(1 - \alpha_2)$ subject to $\alpha_1 + \alpha_2 = \alpha$. Therefore, we need to minimize $\Phi^{-1}(1 - \alpha_1) + \Phi(1 - \alpha + \alpha_1)$.

$$\frac{\partial}{\partial \alpha_1} \Phi^{-1}(1 - \alpha_1) = -\sqrt{2\pi} e^{\frac{z_{\alpha_1}^2}{2}}$$

$$\frac{\partial}{\partial \alpha_1} \Phi^{-1}(1 - \alpha + \alpha_1) = \sqrt{2\pi} e^{\frac{z_{\alpha - \alpha_1}^2}{2}}$$

Upon setting the sum of the two derivatives equal to zero, we obtain $e^{\frac{z_{\alpha - \alpha_1}^2}{2}} = e^{\frac{z_{\alpha_1}^2}{2}}$. This is solved by $z_{\alpha_1} = z_{\alpha - \alpha_1}$. Consequently, $\alpha_1 = \alpha - \alpha_1$, $2\alpha_1 = \alpha$ and $\alpha_1 = \alpha_2 = \frac{\alpha}{2}$.

8.88 a.) $n = 1/2 + (1.9/1)(9.4877/4)$
 $n = 46$

b.) $(10 - .5)/(9.4877/4) = (1 + p)/(1 - p)$
 $p = 0.6004$ between 10.19 and 10.41.

8-89 a)
 $P(X_i \leq \tilde{\mu}) = 1/2$

$$P(\text{all } X_i \leq \tilde{\mu}) = (1/2)^n$$

$$P(\text{all } X_i \geq \tilde{\mu}) = (1/2)^n$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \left(\frac{1}{2}\right)^n + \left(\frac{1}{2}\right)^n - 2\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{n-1}$$

$$1 - P(A \cup B) = P(\min(X_i) < \tilde{\mu} < \max(X_i)) = 1 - \left(\frac{1}{2}\right)^n$$

b.) $P(\min(X_i) < \tilde{\mu} < \max(X_i)) = 1 - \alpha$

The confidence interval is $\min(X_i), \max(X_i)$

8-90 We would expect that 950 of the confidence intervals would include the value of μ . This is due to the definition of a confidence interval.

Let X be the number of intervals that contain the true mean (μ). We can use the large sample approximation to determine the probability that $P(930 < X < 970)$.

$$\text{Let } p = \frac{950}{1000} = 0.950 \quad p_1 = \frac{930}{1000} = 0.930 \quad \text{and} \quad p_2 = \frac{970}{1000} = 0.970$$

$$\text{The variance is estimated by } \frac{p(1-p)}{n} = \frac{0.950(0.050)}{1000}$$

$$\begin{aligned} P(0.930 < p < 0.970) &= P\left(Z < \frac{(0.970 - 0.950)}{\sqrt{\frac{0.950(0.050)}{1000}}} \right) - P\left(Z < \frac{(0.930 - 0.950)}{\sqrt{\frac{0.950(0.050)}{1000}}} \right) \\ &= P\left(Z < \frac{0.02}{0.006892} \right) - P\left(Z < \frac{-0.02}{0.006892} \right) = P(Z < 2.90) - P(Z < -2.90) = 0.9963 \end{aligned}$$