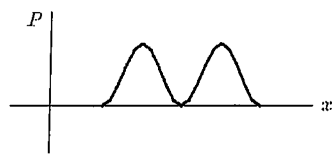
Q.1 For questions (I to VI), circle the correct answer: ***(1 pt each question)***

(I) An electron is in a one-dimensional trap with zero potential energy in the interior and infinite potential energy at the walls. A graph of its probability density P(x) versus x is shown. The value of the quantum number n is:



A. 0 B. 2 C. 1 D. 3 E. 4

(II) An electron is in a one-dimensional well with finite potential energy barriers at the walls. The matter wave:

A. is zero at the barriers
B. is zero in the well
C. extends into the barriers
D. is zero everywhere within each barrier
E. is discontinuous at the barriers

(III) Take the potential energy of a hydrogen atom to be zero for infinite separation of the electron and proton. Then the ground state energy is −13.6 eV. The negative sign indicates:

A. the electron and proton are bound together
B. the potential energy is positive
C. the electron might escape from the atom
D. the kinetic energy is negative
E. none of the above

(IV) Which of the following sets of quantum numbers is possible for an electron in a hydrogen atom?

A. $n = 3, \ell = 1, m_\ell = -2$
B. $n = 4, \ell = 4, m_\ell = -2$
C. $n = 5, \ell = -1, m_\ell = 2$
D. $n = 4, \ell = 3, m_\ell = -3$
E. $n = 2, \ell = 3, m_\ell = -2$

(V) The number of possible values of the magnetic quantum number m_ℓ associated with a given value of the orbital quantum number ℓ is:

A. $\ell/2$ B. $2\ell - 1$ C. $2\ell + 1$ D. 2ℓ E. ℓ

(VI) An electron in a K shell of an atom has the principal quantum number (n=?):

A. 0 B. 1 C. 2 D. 3 E. 4

Q.2 For questions (I to IV), circle the correct answer: ***(1.5 pts each question)***

(I) The ground state energy of an electron in a one-dimensional trap with zero potential energy in the interior and infinite potential energy at the walls is 2.0 eV. If the width of the well is doubled, the ground state energy (in eV) will be:

(A) 0.5 B. 1.0 C. 2.0 D. 4.0 E. 8.0

(II) An electron in an atom initially has an energy 5.5 eV above the ground state energy. It drops to a state with energy 3.2 eV above the ground state energy and emits a photon in the process. The wave associated with the photon has a wavelength (in 10^{-7} m) of:

A. 1.15 B. 3.0 C. 1.7 (D) 5.4 E. 1.0

(III) An electron in an atom is in a state with $\ell = 3$ and $m_\ell = 2$. The semi angle between the orbital angular momentum and the z-axis is:

A. 48.2° (B) 54.7° C. 30.0° D. 35.3° E. 60.0°

(IV) The ratio of the wavelength of the K_α x-ray line for Nb (Z = 41) to that of Ga (Z = 31) is:

A. 3/4 B. 16/9 (C) 9/16 D. 4/3 E. 1.15

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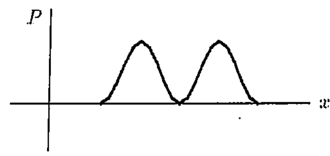

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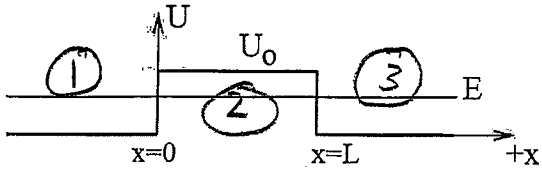
Q.3 A beam of $5\text{x}10^6$-electrons each of energy E = 4 eV originally traveling to the right in a region of space where U = 0 strike a potential barrier (of width L = 0.6 nm and of height U_o = 6 eV) at x=0, as shown in the figure. (Use the notations k_1, k_2 and k_3 for the wave numbers in the regions x < 0, 0<x<L, and x > L, respectively).



(i) Write down the general solution to the time-independent Schrödinger equation for the regions x < 0, 0<x<L, and x > L? ***(3 pts)***

$$\psi_1 = Ae^{ik_1x} + Be^{-ik_2x}, \quad k_1 = \sqrt{2mE}/\hbar$$

$$\psi_2 = Ce^{k_2x} + De^{-k_2x}, \quad k_2 = \sqrt{2m(U_0-E)}/\hbar$$

$$\psi_3 = Fe^{ik_3x}, \quad k_3 = k_1 = \sqrt{2mE}/\hbar$$

(ii) Find the values for the angular wave numbers in the regions x < 0, 0<x<L, and x > L? ***(2 pts)***

$$k_1 = \sqrt{2mE}/\hbar = \sqrt{2\times 9.11\times10^{-31}\times 4\times 1.6\times10^{-19}}\Big/\left(\frac{6.63\times10^{-34}}{2\pi}\right)$$

$$= 1.023\times10^{10}\ (\text{rad/m})$$

$$k_2 = \sqrt{2m(U_0-E)}/\hbar = \sqrt{2\times9.11\times10^{-31}\times(6-4)\times1.6\times10^{-19}}\Big/\left(\frac{6.63\times10^{-34}}{2\pi}\right)$$

$$= 0.724\times10^{10}\ (\text{rad/m})$$

$$k_3 = k_1 = 1.023\times10^{10}\ (\text{rad/m})$$

(iii) Find the number of electrons that are expected to tunnel through the potential barrier? ***(2 pts)***

$$T \approx e^{-2k_2L} = e^{-2L\sqrt{2m(U_0-E)}/\hbar}$$

$$\approx e^{-2\times0.6\times10^{-9}\times0.724\times10^{10}} \simeq 1.686\times10^{-4}$$

No. ~~of transmitted~~ electrons = T * No. of incidet electrons

$$= 1.686\times10^{-4}\times5\times10^{6} \simeq 843\ (\text{electrons})$$

Q.4 An electron in the hydrogen atom is in the state $\psi_{3,2,-1}$.

(i) Find the magnitude of the electron's orbital magnetic dipole moment in this state? ***(1.5 pts)***

$$\mu_{orb.} = \frac{e}{2m}\sqrt{l(l+1)}\hbar = \sqrt{l(l+1)}\,\mu_B$$
$$= \sqrt{2(2+1)}\; 9.274\times10^{-24} = 2.272\times10^{-23}\ (J/T)$$

(ii) Find the z-component of the electron's orbital magnetic dipole moment in this state? ***(1.5 pts)***

$$\mu_{orb.,z} = -m_l\,\mu_B = -(-1)\,9.274\times10^{-24} = +9.274\times10^{-24}\,(J/T)$$

(iii) Find the z-component of the electron's spin magnetic dipole moment, if it is in a *"spin up"* state? ***(1.5 pts)***

$$\mu_{s,z} = -2m_s\mu_B = -2\left(\tfrac{1}{2}\right)(9.274\times10^{-24}) = -9.274\times10^{-24}\,(J/T)$$

(iv) Find the required energy to ionize the hydrogen atom when it is in the state $\psi_{3,2,-1}$? ***(1.5 pts)***

$$E_{ionization} = |E_{Bound}| = |E_3| = \left|\frac{-13.6}{(3)^2}\right|$$
$$= 1.51\,(eV) \simeq 2.418\times10^{-19}\,(J)$$