Q.1 For questions (I to VII), circle the correct answer: *(1 pt each question)*

(I) An electron is in a one-dimensional trap with zero potential energy in the interior and infinite potential energy at the walls. The ratio E_3/E_1 of the energy for n = 3 to that for n = 1 is:

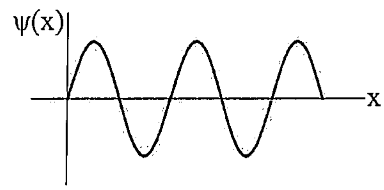
a. 1/4 b. 1/9 c. 3/1 d. 9/1 e. 4/1

(II) The energy of the third excited state of "an electron in a one-dimensional trap with zero potential energy in the interior and infinite potential energy at the walls" is 32.0 eV. The ground state energy will be:

a. 4.0 eV b. 8.0 eV c. 2.0 eV d. 3.0 eV e. None of them

(III) An electron is in a one-dimensional trap with zero potential energy in the interior and infinite potential energy at the walls. A graph of its wave function $\psi(x)$ versus x is shown. The value of the quantum number n is:

a. 4 b. 2 c. 8 d. 6 e. 5



(IV) If a wave function ψ for a particle moving along the x axis is normalized, then:

a. $\int |\psi|^2\, dt = 1$ b. $\int |\psi|^2\, dx = 1$ c. $\partial\psi/\partial x = 1$ d. $\partial\psi/\partial t = 1$ e. $|\psi|^2 = 1$

(V) For the hydrogen atom an electron is in the state (with the orbital quantum number, $l = 3$), the number of different allowed values for the magnetic quantum number m_l is equal to:

a. 7 b. 6 c. 3 d. 4 e. none of them

(VI) When a beam of electrons of kinetic energy 40 keV strike a molybdenum target, they produce both continuous and characteristic x-ray. If we increase the kinetic energy of the electrons to 80 keV, then the energy of K_α line will:

a. Be doubled b. Be half the original value c. not change

d. Be four times the original value e. Be decreased to one fourth the original value

(VII) In atomic states a metastable state is a state with:

a. Life time much smaller than other states b. Life time much larger than other states

c. Life time about that of other states d. none of them

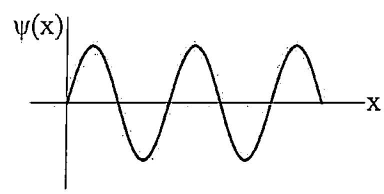
Q.1 For questions (I to VII), circle the correct answer: ***(1 pt each question)***

(I) An electron is in a one-dimensional trap with zero potential energy in the interior and infinite potential energy at the walls. The ratio E_1/E_3 of the energy for n = 1 to that for n = 3 is:

a. 1/4 (b.) 1/9 c. 3/1 d. 9/1 e. 4/1

(II) The energy of the third excited state of "an electron in a one-dimensional trap with zero potential energy in the interior and infinite potential energy at the walls" is 64.0 eV. The ground state energy will be:
(a.) 4.0 eV b. 8.0 eV c. 2.0 eV d. 3.0 eV e. None of them

(III) An electron is in a one-dimensional trap with zero potential energy in the interior and infinite potential energy at the walls. A graph of its wave function ψ(x) versus x is shown. The value of the quantum number n is:
a. 4 b. 2 (c.) 5 d. 6 e. 8



(IV) If a wave function ψ for a particle moving along the x axis is normalized, then:

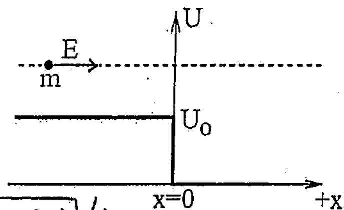
a. $\int |\psi|^2 \, dt = 1$ b. $\partial\psi/\partial t = 1$ c. $\partial\psi/\partial x = 1$ (d.) $\int |\psi|^2 \, dx = 1$ e. $|\psi|^2 = 1$

(V) For the hydrogen atom an electron is in the state (with the orbital quantum number, $l = 4$), the number of different allowed values for the magnetic quantum number m_l is equal to:
a. 5 b. 8 (c.) 9 d. 4 e. none of them

(VI) When a beam of electrons of kinetic energy 40 keV strike a molybdenum target, they produce both continuous and characteristic x-ray. If we increase the kinetic energy of the electrons to 80 keV, then the energy of K_α line will:
a. Be doubled b. Be half the original value c. Be four times the original value
d. Be decreased to one fourth the original value (e.) not change

(VII) In atomic states a metastable state is a state with:
(a.) Life time much larger than other states b. Life time much smaller than other states
c. Life time about that of other states d. none of them

Q.2 An electron of energy E = 8 eV originally traveling to the right in a region of space where U_o = 6 eV (for x<0) strike "a step down potential of U=0 (for x>0)" at x=0, as shown in the figure. (Use the notations k_1 and k_2 for the wave numbers in the regions x < 0 and x > 0, respectively).



(i) Write down the general solution to the time-independent Schrödinger equation for the regions x < 0 and x > 0? *(2 pts)*

For $x<0$: $\psi_1 = Ae^{ik_1x} + Be^{-ik_1x}$, $k_1 = \sqrt{2m(E-U_0)}/\hbar$

For $x>0$: $\psi_2 = Ce^{ik_2x}$, $k_2 = \sqrt{2mE}/\hbar$

(ii) Use the boundary conditions on the wave function to write linear equations for the unknown constants of your general solution of part (i)? *(2 pts)*

$\psi_1|_{x=0} = \psi_2|_{x=0} \Rightarrow A + B = C$ --- ①

$\psi_1'|_{x=0} = \psi_2'|_{x=0} \Rightarrow (A-B)k_1 = Ck_2$ --- ②

① $*k_1$ + ② $\Rightarrow 2Ak_1 = C(k_1+k_2) \rightarrow \frac{C}{A} = \frac{2k_1}{k_1+k_2}$

① $*k_1$ − ② $\Rightarrow +2Bk_1 = C(k_1-k_2) = \frac{2Ak_1(k_1-k_2)}{(k_1+k_2)}$

$\frac{B}{A} = \frac{k_1-k_2}{k_1+k_2}$

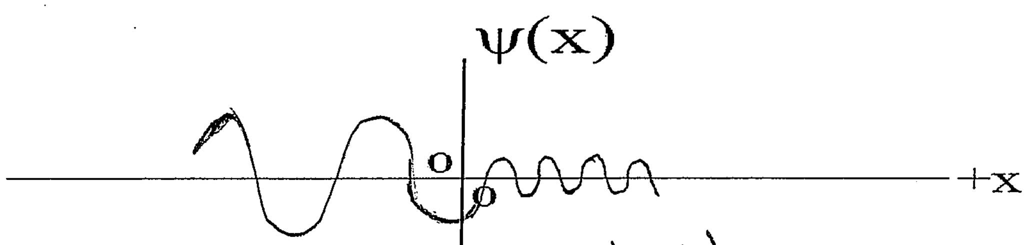
(iii) Find the values for the wave numbers in the regions x < 0 and x > 0? *(1 pts)*

$k_1 = \sqrt{2m_e(E-U_0)}/\hbar = \sqrt{2\times 9.11\times10^{-31}\times(8-6)\times1.6\times10^{-19}} / \left(\frac{6.63\times10^{-34}}{2\pi}\right)$

$\cong 7.24\times10^{9}$ (rad/m)

$k_2 = \sqrt{2mE}/\hbar = \sqrt{2\times9.11\times10^{-31}\times8\times1.6\times10^{-19}} / \left(\frac{6.63\times10^{-34}}{2\pi}\right) \cong 14.47\times10^{9}$ (rad/m)

(iv) Sketch the solution, using the x-axis in the following figure. *(2 pts)*



$\lambda_2 < \lambda_1$

(Amplitude) in ② < (Amplitude) in ①

(v) The reflection coefficient is R = $[(k_2 - k_1)/ (k_2 + k_1)]^2$, if there are 1000-electrons incident, find the number of transmitted electrons? *(2 pts)*

$T = 1 - R = 1 - \left[\frac{k_2-k_1}{k_2+k_1}\right]^2 = 1 - \left[\frac{14.47-7.24}{14.47+7.24}\right]^2 \cong 0.889$

$T \cong 88.9$

Number of transmited electrons $= (0.889)*(1000)$

$= 889$ (electrons)

Q.3 An electron in the hydrogen atom is in the state $\psi_{1,0,0} = A\, e^{-r/a}$.

(i) Normalize this wave function (i.e find the value for A that will make the wave function normalized) *(2 pts)*

$\int \psi \psi^* dV = 1$, since ψ depends on r only, then $dV = 4\pi r^2 dr$

$$\int_0^\infty A^2 e^{-2r/a} 4\pi r^2 dr = 1 \rightarrow 4\pi A^2 \int_0^\infty r^2 e^{-2r/a} dr = 1$$

$$4\pi A^2 \left[\frac{2!}{(2/a)^3}\right] = \pi A^2 a^3 = 1 \rightarrow A = \frac{1}{\sqrt{\pi}\, a^{3/2}}$$

(ii) Find the probability that the electron in this state will be found between two spherical shells with radii 0.8a and 1.2a? *(2 pts)*

$$P = \int_{r=0.8a}^{1.2a} \psi^2 4\pi r^2 dr = \frac{4\pi}{\pi a^3} \int_{0.8a}^{1.2a} r^2 e^{-2r/a} dr = \frac{4}{a^3}\left[\frac{-1}{\left(\frac{2}{a}\right)^3}\left(\left(\frac{2}{a}\right)^2 r^2 + 2\frac{2}{a} r + 2\right) e^{-\frac{2}{a}r}\right]_{r=0.8}^{1.2a}$$

$$= -\frac{1}{2}\left[\frac{4}{a^2}r^2 + \frac{4}{a}r + 2\right] e^{-2r/a}\Big|_{0.8a}^{1.2a} = -2\left[\frac{r^2}{a^2} + \frac{r}{a} + 0.5\right] e^{-2r/a}\Big|_{0.8a}^{1.2a}$$

$$= 2\left[-\frac{(1.2)^2 a^2}{a^2} + \frac{1.2a}{a} + 0.5\right] e^{\frac{-2(1.2a)}{a}} + \frac{(0.8)^2 a^2}{a^2} + \frac{0.8a}{a} + 0.5\, e^{\frac{-2(0.8a)}{a}}\Big]$$

$$= 2\left[-0.28122 + 0.3917\right] \cong 0.214 \cong 21.4\%$$

Q.4 A rectangular corral of widths $L_x = L$ and $L_y = L/2$ contains seven non interacting electrons with spin of $s = ½$ for each electron.

(i) Write down the general form for the energy of any level. *(2 pts)*

$$E_{n_x,n_y} = n_x^2 \frac{h^2}{8mL_x^2} + n_y^2 \frac{h^2}{8mL_y^2} = n_x^2 \frac{h^2}{8mL^2} + n_y^2 \frac{h^2}{8mL^2/4}$$

$$= (n_x^2 + 4n_y^2)\frac{h^2}{8mL^2}, \quad n_x = 1, 2, 3, \ldots \quad n_y = 1, 2, 3, \ldots$$

(i) Find the energy of the ground state of this system of seven electrons? *(3 pts)*

$E_{1,1} = 5\, h^2/8mL^2$

$E_{2,1} = 8\, h^2/8mL^2$

$E_{3,1} = 13\, h^2/8mL^2$

$E_{1,2} = 17\, h^2/8mL^2$

$$E_{ground} = 2E_{1,1} + 2E_{2,1} + 2E_{3,1} + 1E_{1,2}$$

$$= \left[2(5) + 2(8) + 2(13) + 1(17)\right]\frac{h^2}{8mL^2}$$

$$= 69\left[\frac{h^2}{8mL^2}\right]$$