SQU Department of Mathematics & Statistics
MATH 2108: Calculus II Fall 2010
Homework Assignment

To submit no later than Saturday, 4 December, 2010

- This assignment carries a 5% weight of the total course weight
- Present a clear, detailed and thought out work
- Your work must be independently executed
- All cases of plagiarism, if detected, will be dealt with as per university exam regulation
- An entire question or a part of it may be assessed by a quiz during the class
- Show all necessary work to receive full credit. Simplify your answer as far as possible.

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1. Let R be the triangle enclosed by three lines $y = \frac{1}{2}x - 1$, $y = -\frac{1}{2}x + 1$ and $x = 4$.
   (*a*) Sketch the region R and find the area of R.
   (*b*) Find the volume V obtained by rotating R about the y-axis.

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2. (*a*) Find the area between $y = x^2$ and $y = mx$ for any constant $m > 0$.
   (*b*) Using the area found in (*a*), and without doing any further calculations, find the area between $y = \sqrt{x}$ and $y = mx$.

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3. The base of a solid V is the region bounded by $x + y = 2$, $y = \ln(x-1)$ and $y = 2$.
   Find the volume of this solid if V has the following cross sections perpendicular to the y-axis:
   (*a*) square cross sections
   (*b*) semicircular cross sections
   (*c*) equilateral triangle cross sections.

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4. Evaluate the following: [*Use a table of integration if needed*]

   (*a*) $\int \frac{4-2x^2}{x^3+3x^2+2x}\,dx$; (*b*) $\int x^{-4}\sin(1/x)\cos(1/x)\,dx$;

   (*c*) $\int \frac{\ln x\sqrt{\ln^4 x - 9}}{5x}\,dx$; (*d*) $\int_{-1}^{1} \frac{2}{13-4|x|+x^2}\,dx$.

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5. Use a comparison to determine whether the integral converges or diverges:

   (*a*) $\int_3^\infty \frac{1}{x-e^{-x}}\,dx$ (*b*) $\int_3^\infty \frac{1}{x+e^{x}}\,dx$ (*c*) $\int_2^\infty \frac{\ln x}{e^x+1}\,dx$ (*d*) $\int_1^\infty e^{-x^3}\,dx$

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6. Determine whether TRUE or FALSE and justify your answer: (*where appropriate give a counter example*)

   (*a*) If $\lim_{x\to\infty} f(x) = 0$ then $\int_0^\infty f(x)\,dx$ converges.

   (*b*) For all integers $M \geq 2$, $\left|\sum_{k=1}^{\infty}(-1)^k \frac{1}{\sqrt{k}} - \sum_{k=1}^{M-1}(-1)^k \frac{1}{\sqrt{k}}\right| \leq \frac{1}{\sqrt{M}}$.

   (*c*) If a_n is the area enclosed by $y = x^{2n}$ and $y = x^{1/2n}$ for $n \geq 1$, $x \in [0,1]$, then $\lim_{n\to\infty} a_n = 1$.

   (*d*) If $\sum_{k=1}^{\infty} a_k$ diverges then $\lim_{k\to\infty} a_k \neq 0$.

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7. Let $a_n = \frac{1}{n^3} + \frac{4}{n^3} + \frac{9}{n^3} + \cdots + \frac{n^2}{n^3}$.

   (*a*) Find a closed form for a_n.

   (*b*) Show that the sequence a_n converges.

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8. Determine whether the series converges or diverges:

   (*a*) $\sum_{k=1}^{\infty} \tan\left(e^{-k}\right)$ (*b*) $\sum_{k=1}^{\infty} \frac{2k+1}{k^2(k+1)^2}$ (*c*) $\sum_{k=3}^{\infty} \frac{k^{-1}}{\ln k + 1}$

   (*d*) $\sum_{k=1}^{\infty} \frac{1}{k\sqrt{1+3k}}$ (*e*) $\sum_{k=1}^{\infty} \frac{1}{|\sin k| + k\sqrt{k}}$ (*f*) $\sum_{k=3}^{\infty} \frac{\cos k\pi}{k + \ln k}$

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